Learning unit 2

MOMENT OF A FORCE
and
MOMENT OF A COUPLE

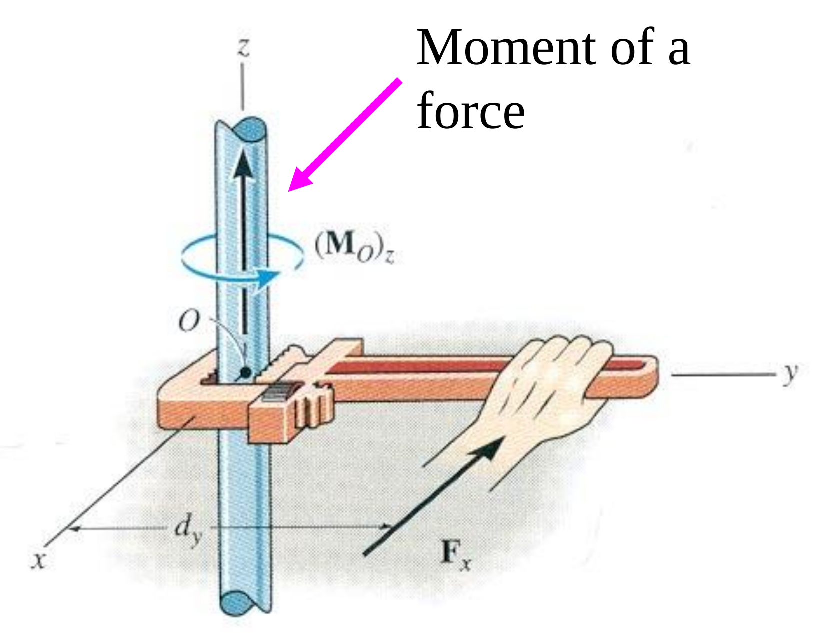
MOMENT OF A FORCE

Today's Objectives :

Students will be able to:

a) understand and define moment, and,

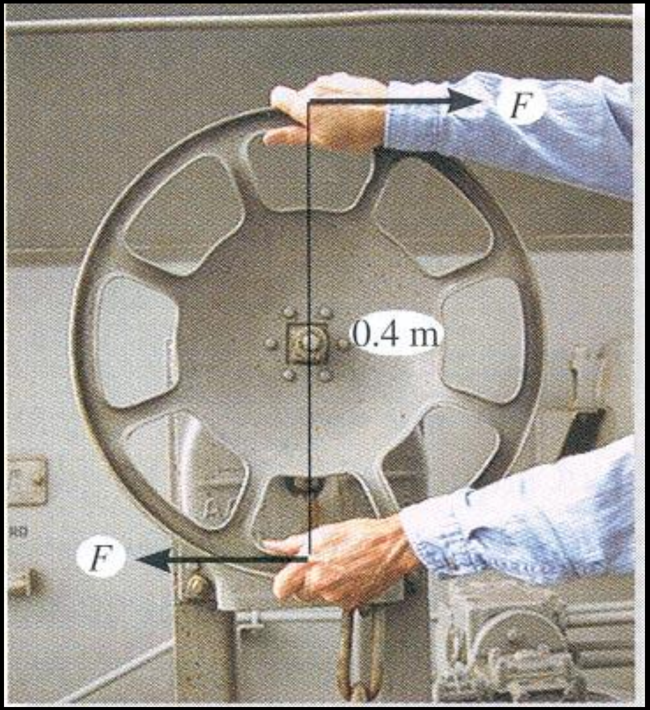
b) determine moments of a force in 2-D and 3-D cases.



In-Class Activities :

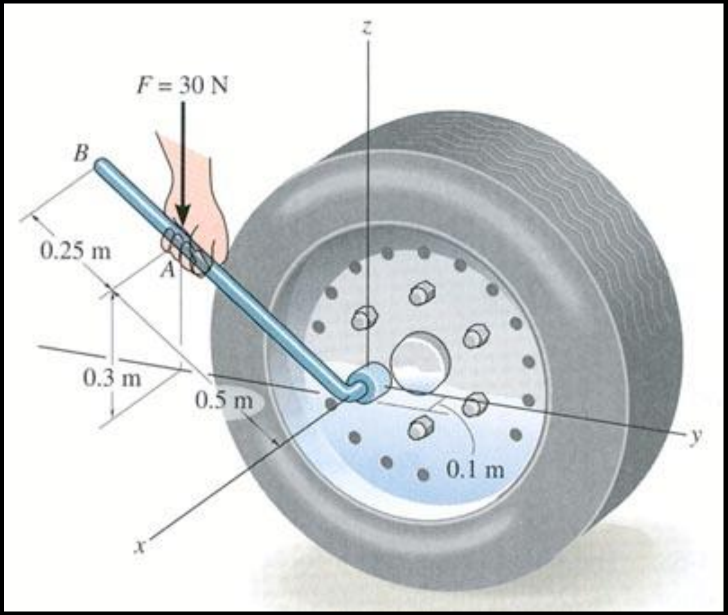
- Check homework, if any
- Reading quiz
- Applications
- Moment in 2-D
- Moment in 3-D
- Concept quiz
- Group Problem Solving
- Attention quiz

APPLICATIONS



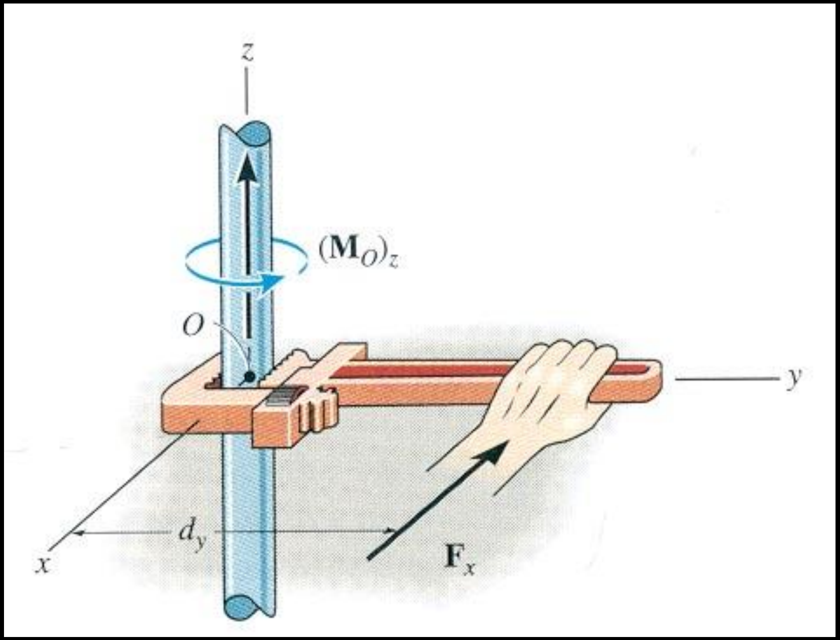
What is the net effect of the two forces on the wheel?

APPLICATIONS (continued)



What is the effect of the 30 N force on the lug nut?

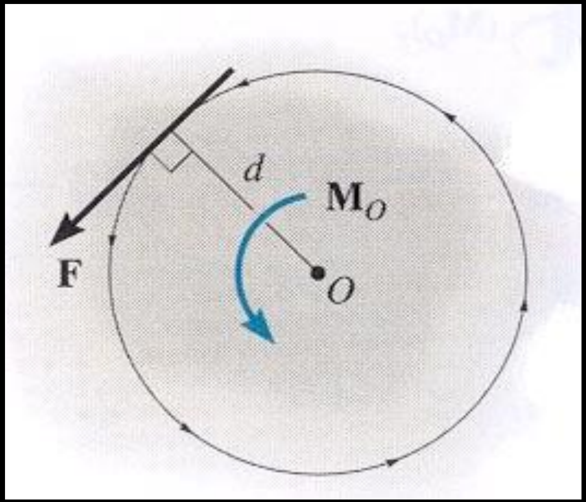
MOMENT IN 2-D



The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

MOMENT IN 2-D (continued)

In the 2-D case, the magnitude of the moment is
$M_o = F\ d$

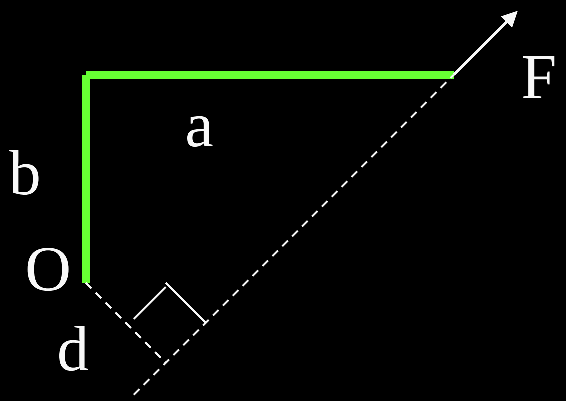


As shown, d is the *perpendicular* distance from point O to the line of action of the force.

In 2-D, the direction of M_O is either clockwise or

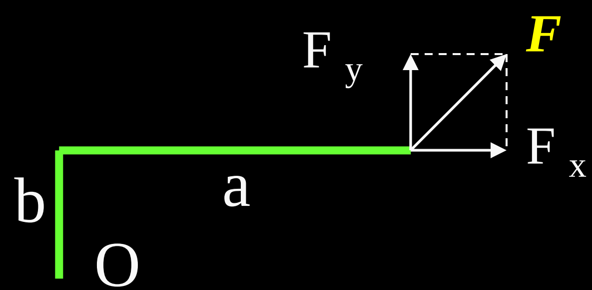
counter-clockwise depending on the tendency for rotation.

MOMENT IN 2-D (continued)



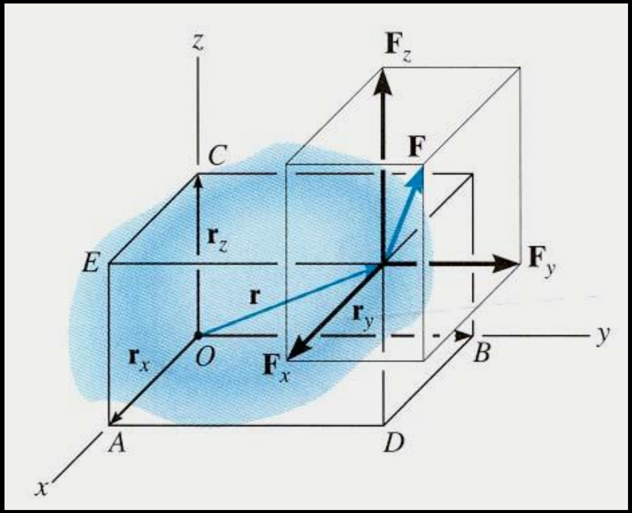
For example, M_O = F d and the direction is counter-clockwise.

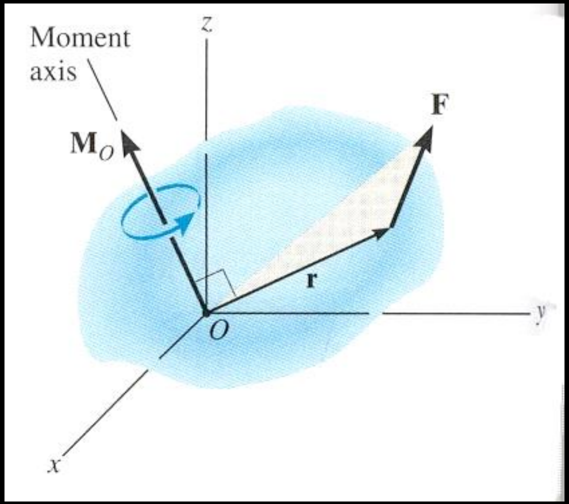
Often it is easier to determine M_O by using the components of ***F*** as shown.



Using this approach, $M_O = (F_Y\ a) – (F_X\ b)$. Note the different signs on the terms! The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive. We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

MOMENT IN 3-D (Vector formulation)



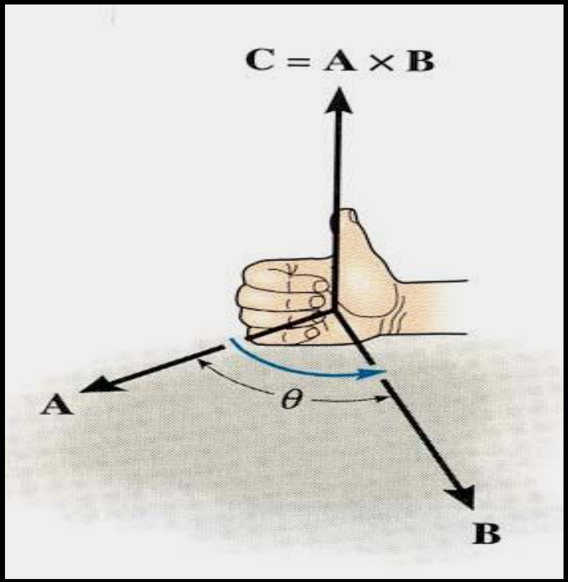


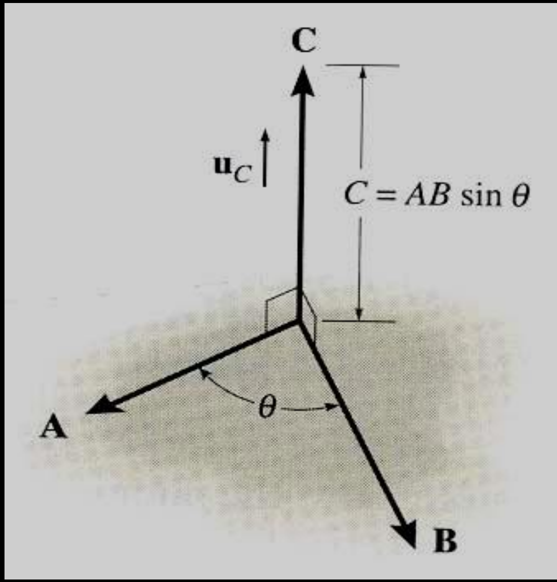
Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$.

Here $\boldsymbol{r}$ is the position vector from point O to <u>any point</u> on the line of action of $\boldsymbol{F}$.

CROSS PRODUCT





In general, the cross product of two vectors ***A*** and ***B*** results in another vector ***C*** , i.e., ***C = A × B***. The magnitude and direction of the resulting vector can be written as

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \mathrm{A\,B} \sin \theta \, \boldsymbol{U_C}$$

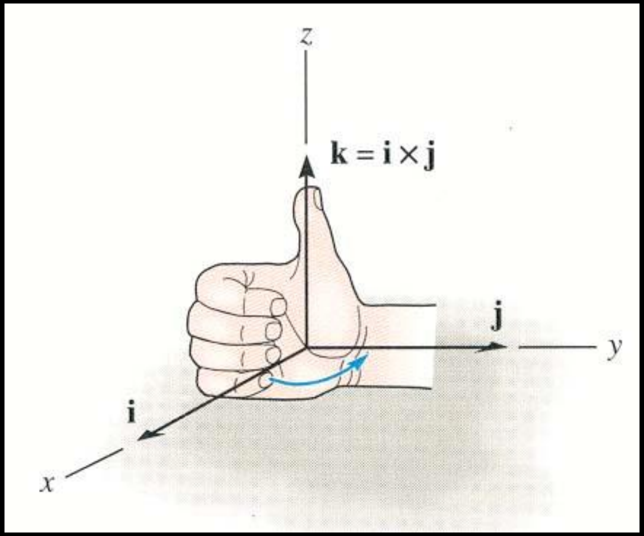
Here $\boldsymbol{U_C}$ is the unit vector perpendicular to both A and B vectors as shown (or to the plane containing the A and B vectors).

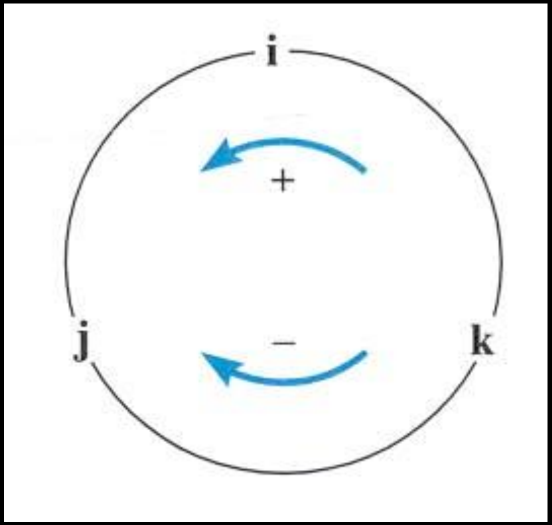
CROSS PRODUCT

The right hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$

Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$



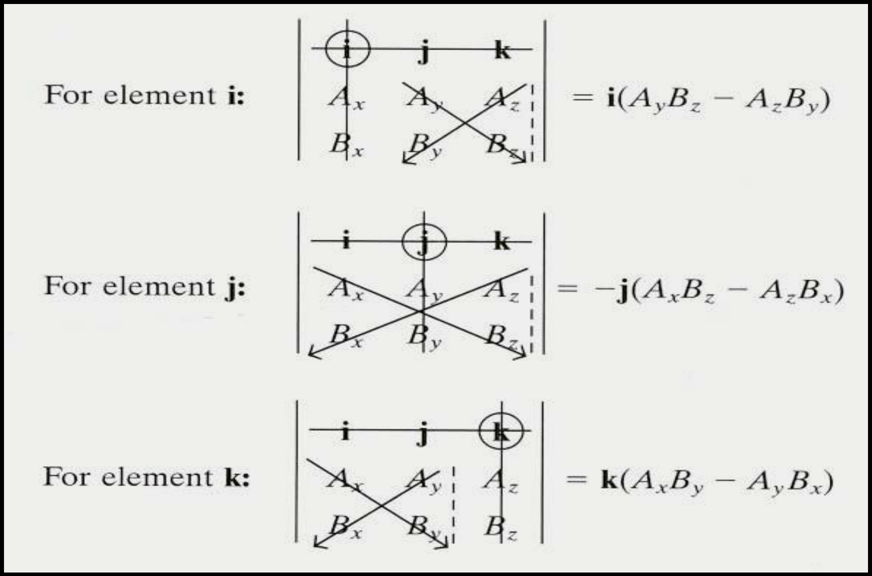


CROSS PRODUCT (continued)

Of even more utility, the cross product can be written as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.



MOMENT IN 3-D (continued)

So, using the cross product, a moment can be expressed as

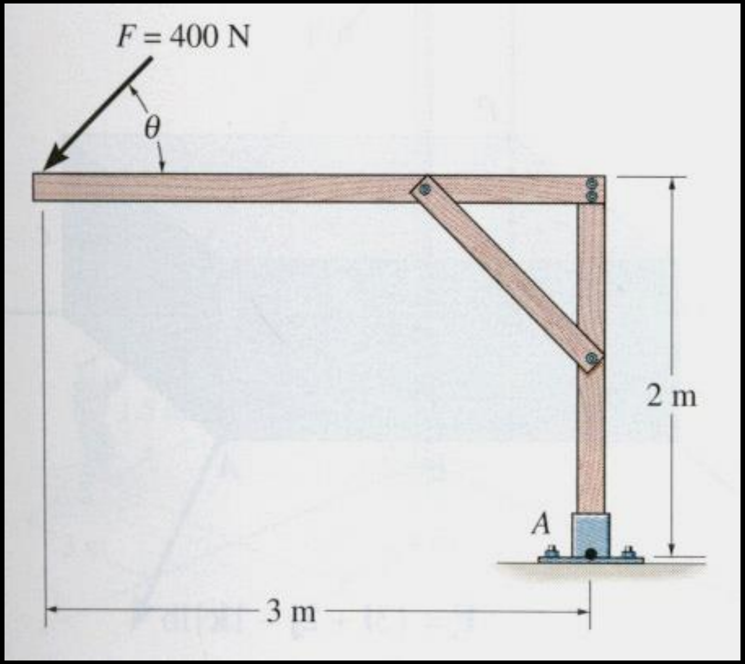
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2 × 2 determinants (see Section 4.2), we get (sample units are N - m or lb - ft)

$$\boldsymbol{M_O} = (r_y\ F_Z - r_Z\ F_y)\ \boldsymbol{i} - (r_x F_z - r_z F_x)\ \boldsymbol{j} + (r_x F_y - r_y F_x)\ \boldsymbol{k}$$

The physical meaning of the above equation becomes evident by considering the force components separately and using a 2-D formulation.

EXAMPLE 1



Given: A 400 N force is applied to the frame and θ = 20°.

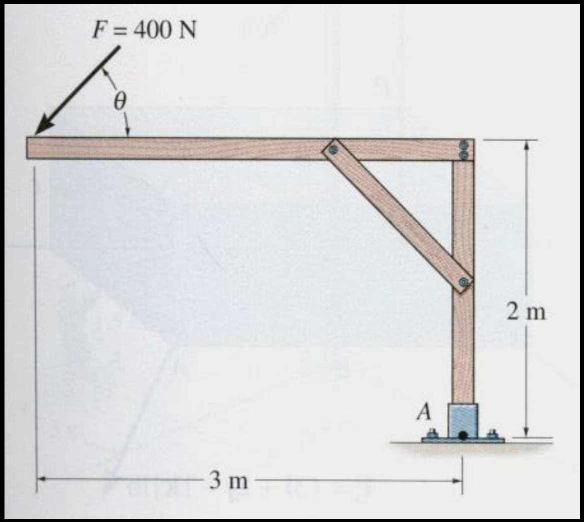
Find: The moment of the force at A.

Plan:

1) Resolve the force along x and y axes.

2) Determine M_A using scalar analysis.

EXAMPLE 1 (continued)



<u>Solution</u>

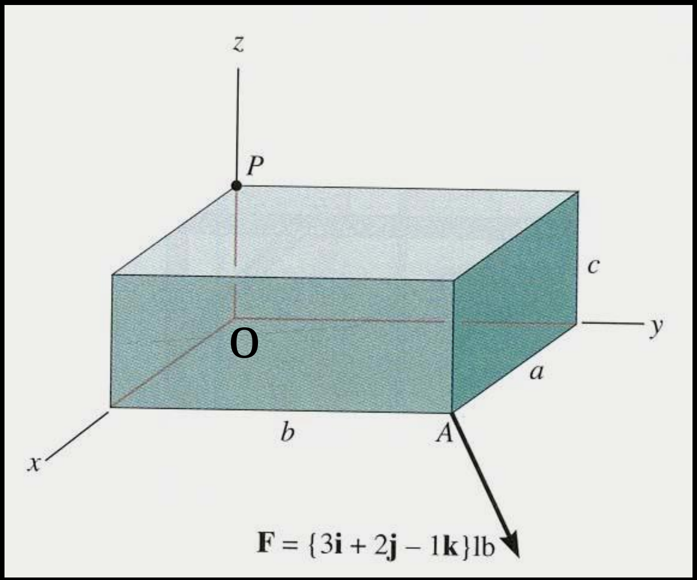
$+\uparrow F_y = -400 \cos 20° \text{ N}$

$+\rightarrow F_x = -400 \sin 20° \text{ N}$

$\circlearrowleft + M_A = \{(400 \cos 20°)(2) + (400 \sin 20°)(3)\} \text{ N·m}$

$= 1160 \text{ N·m}$

EXAMPLE 2



Given: a = 3 in, b = 6 in and c = 2 in.

Find: Moment of $\boldsymbol{F}$ about point O.

Plan:

1) Find $\boldsymbol{r}_{OA}$.

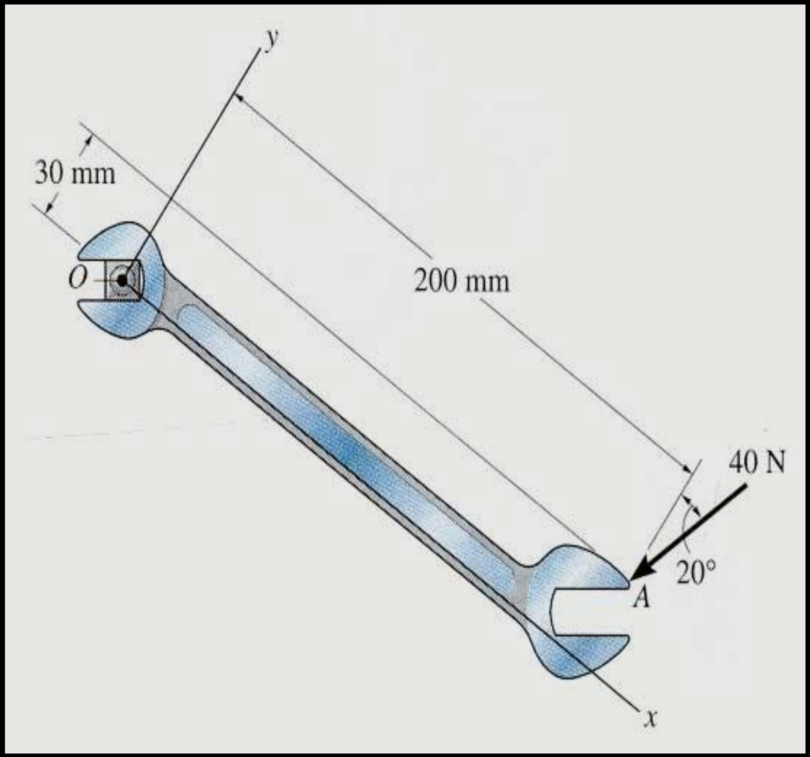
2) Determine $\boldsymbol{M}_O = \boldsymbol{r}_{OA} \times \boldsymbol{F}$.

Solution $\boldsymbol{r}_{OA} = \{3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 0\,\boldsymbol{k}\}$ in

$$M_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & 0 \\ 3 & 2 & -1 \end{vmatrix} = [\{6(-1) - 0(2)\}\,\boldsymbol{i} - \{3(-1) - 0(3)\}\,\boldsymbol{j} + \{3(2) - 6(3)\}\,\boldsymbol{k}]\ \text{lb·in}$$

$$= \{-6\,\boldsymbol{i} + 3\,\boldsymbol{j} - 12\,\boldsymbol{k}\}\ \text{lb·in}$$

GROUP PROBLEM SOLVING



Given: A 40 N force is applied to the wrench.

Find: The moment of the force at O.

Plan: 1) Resolve the force along x and y axes.

2) Determine M_O using scalar analysis.

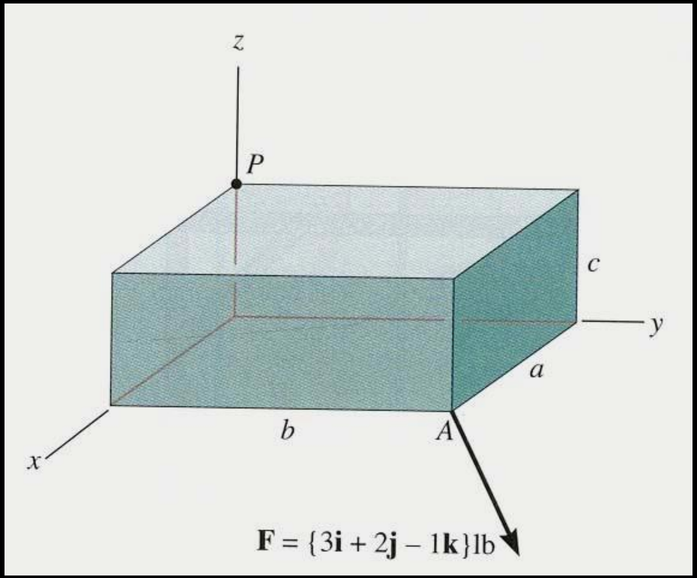
Solution: $+\uparrow F_y = -40 \cos 20° \text{ N}$

$+\rightarrow F_x = -40 \sin 20° \text{ N}$

$$\circlearrowleft + \quad M_O = \{-(40 \cos 20°)(200) + (40 \sin 20°)(30)\} \text{N·mm}$$

$$= -7107 \text{ N·mm} = -7.11 \text{ N·m}$$

GROUP PROBLEM SOLVING



Given: a = 3 in , b = 6 in and c = 2 in

Find: Moment of F about point P

Plan: 1) Find $\boldsymbol{r_{PA}}$.

2) Determine $\boldsymbol{M_P} = \boldsymbol{r_{PA}} \times \boldsymbol{F}$

.

Solution: $\boldsymbol{r}_{PA} = \{\, 3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 2\,\boldsymbol{k} \,\}$ in

$$M_P = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & -2 \\ 3 & 2 & -1 \end{vmatrix} = \{ -2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 12\,\boldsymbol{k} \} \text{ lb} \cdot \text{in}$$

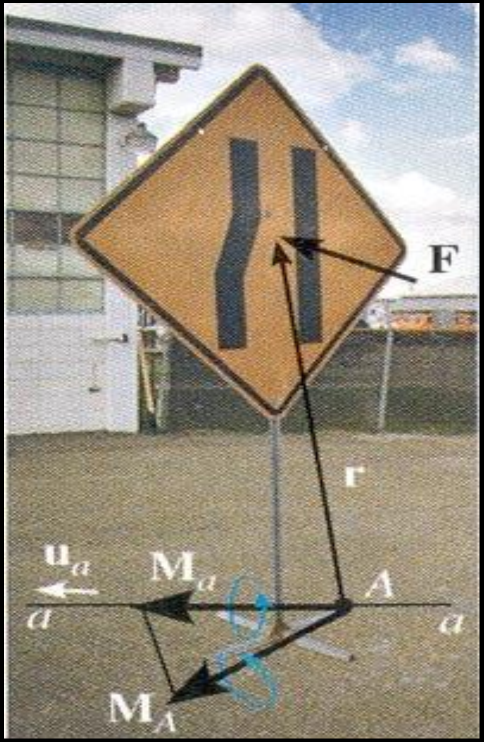
MOMENT ABOUT AN AXIS

Today's Objectives:

Students will be able to determine the moment of a force about an axis using

a) scalar analysis, and

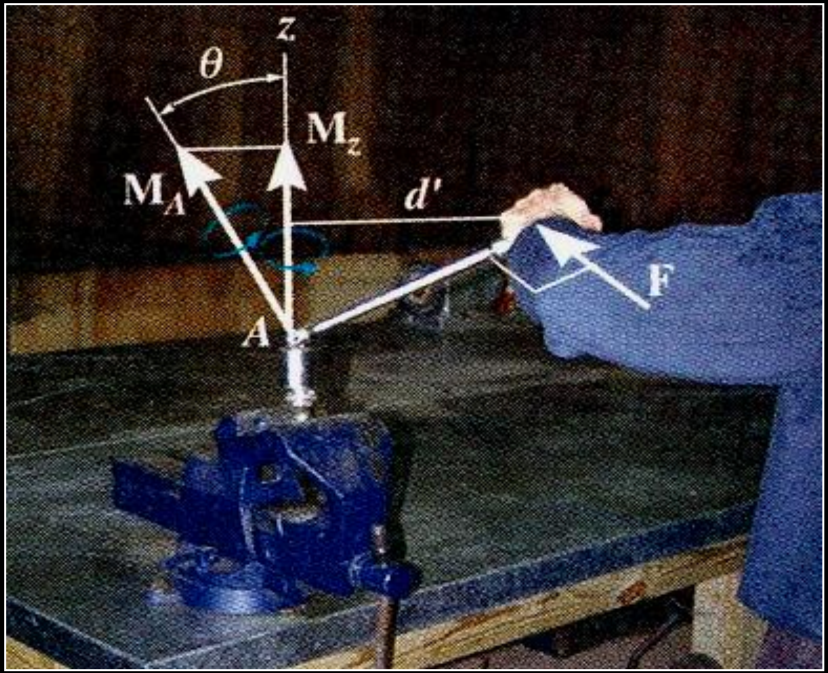
b) vector analysis.

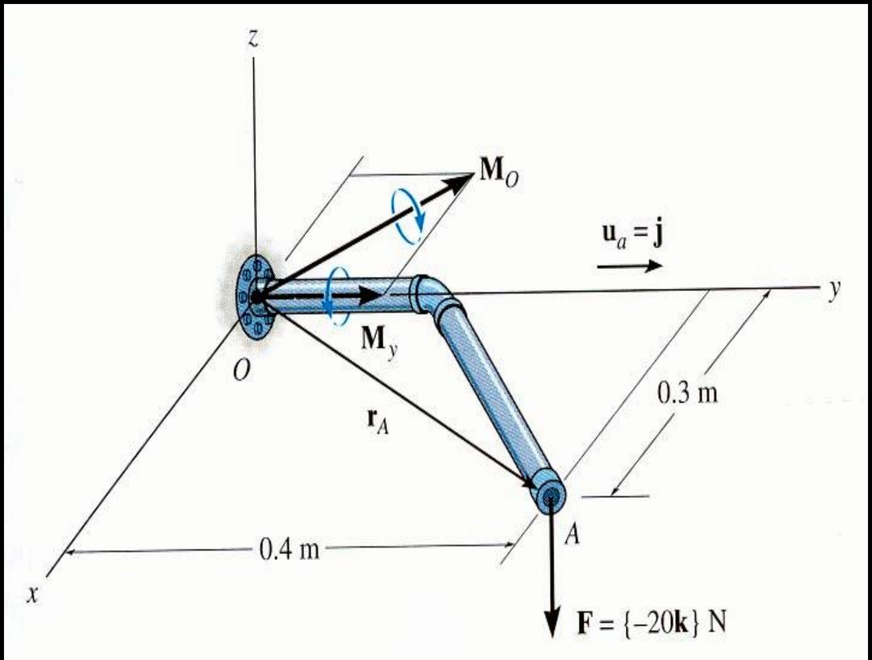


In-Class Activities:

- Check Home work, if any
- Reading quiz
- Applications
- Scalar analysis
- Vector analysis
- Concept quiz
- Group problem solving
- Attention quiz

APPLICATIONS



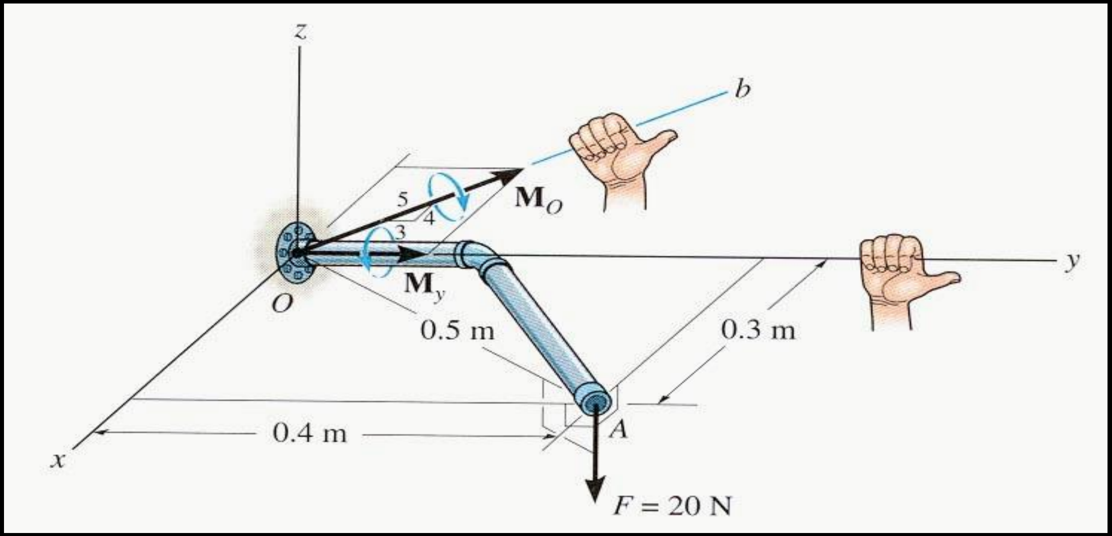


With the force ***F***, a person is creating the moment M_A. What portion of M_A is used in turning the socket?

The force ***F*** is creating the moment M_O. How much of M_O acts to unscrew the pipe?

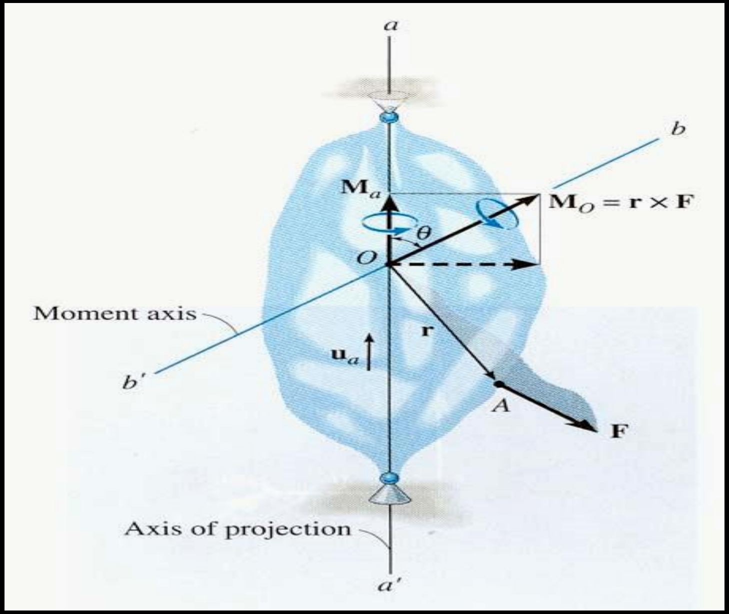
SCALAR ANALYSIS

Recall that the moment of a force about any point A is $M_A = F\, d_A$ where d_A is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.



In the figure above, the moment about the y-axis would be $M_y = 20\ (0.3) = 6$ N·m. However this calculation is not always trivial and vector analysis may be preferable.

VECTOR ANALYSIS



Our goal is to find the moment of ***F*** (the tendency to rotate the body) about the axis a'-a.

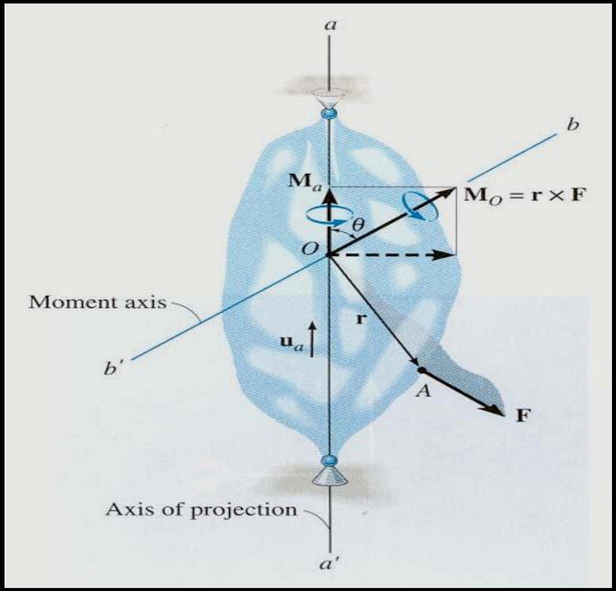
First compute the moment of ***F*** about any arbitrary point O that lies on the a'a axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

Now, find the component of $\boldsymbol{M_O}$ along the axis a'-a using the dot product.

$$\boldsymbol{M_a} = \boldsymbol{u_a} \bullet \boldsymbol{M_O}$$

VECTOR ANALYSIS (continued)



M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the triple scalar product.

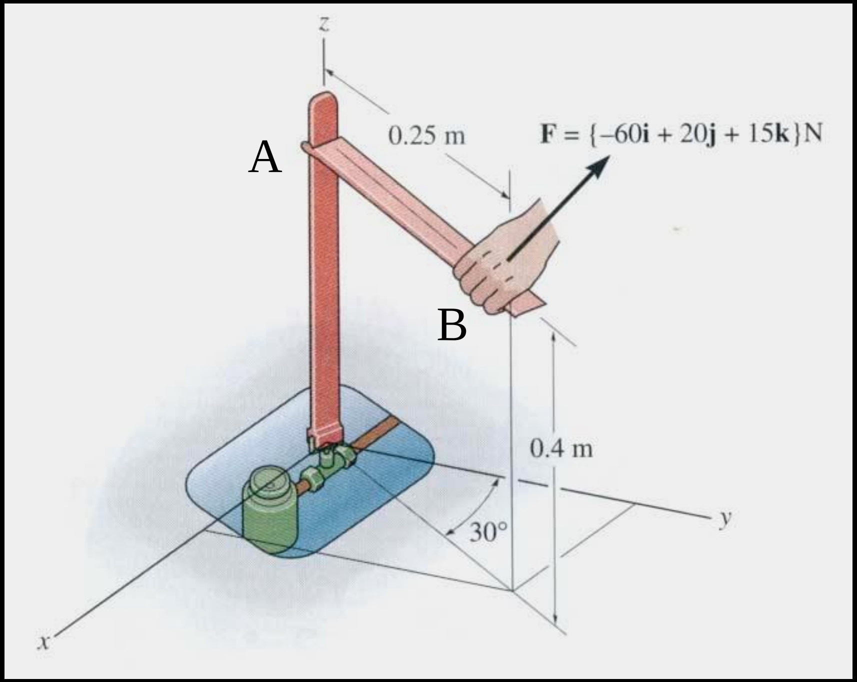
In the this equation,

u_a represents the unit vector along the axis a'-a axis,

r is the position vector from any point on the a'-a axis to any point A on the line of action of the force, and

F is the force vector.

EXAMPLE



Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

Plan:

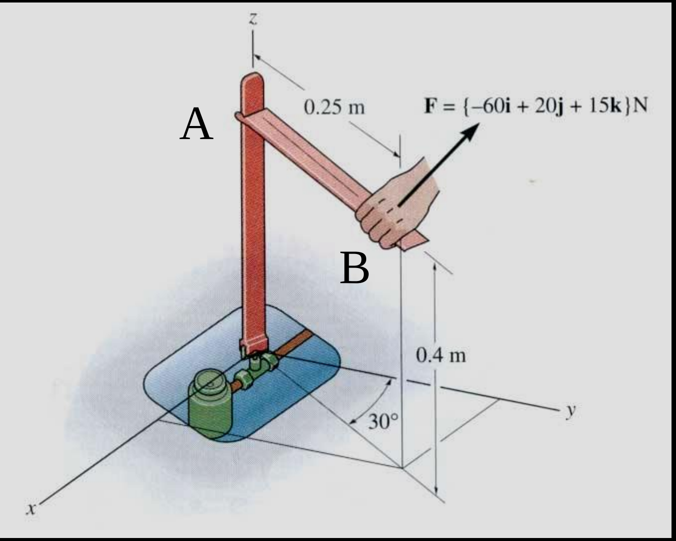
1) We need to use $M_z = u \cdot (r \times F)$.

2) Note that $u = 1\,k$.

3) The vector r is the position vector from A to B.

4) Force F is already given in Cartesian vector form.

EXAMPLE (continued)



$\boldsymbol{u} = 1\,\boldsymbol{k}$

$\boldsymbol{r_{AB}} = \{0.25 \sin 30°\, \boldsymbol{i} + 0.25 \cos 30°\, \boldsymbol{j}\}$ m

$= \{0.125\, \boldsymbol{i} + 0.2165\, \boldsymbol{j}\}$ m

$\boldsymbol{F} = \{-60\, \boldsymbol{i} + 20\, \boldsymbol{j} + 15\, \boldsymbol{k}\}$ N

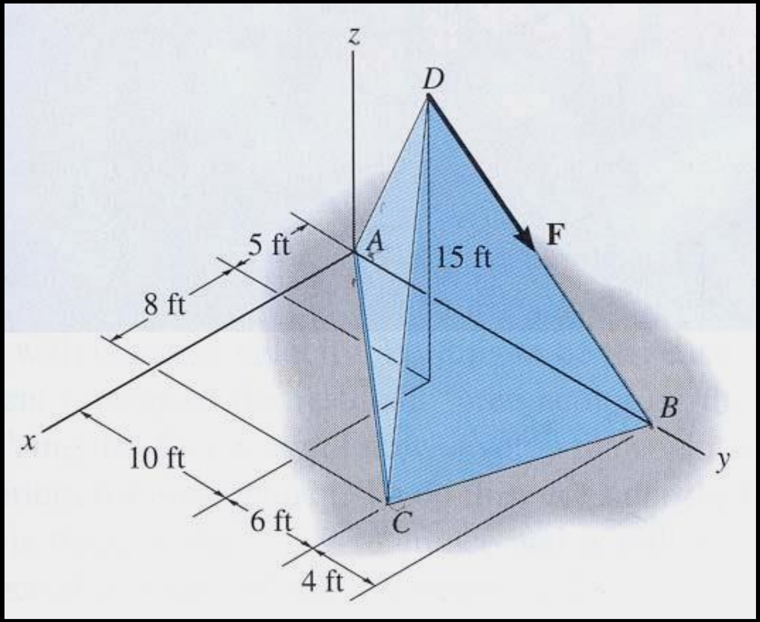
$M_z = \boldsymbol{u} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.2165 & 0 \\ -60 & 20 & 15 \end{vmatrix}$$

$= 1\{0.125(20) - 0.2165(-60)\}$ N·m

$= 15.5$ N·m

GROUP PROBLEM SOLVING



Given: A force of 80 lb acts along the edge DB.

Find: The magnitude of the moment of this force about the axis AC.

Plan:

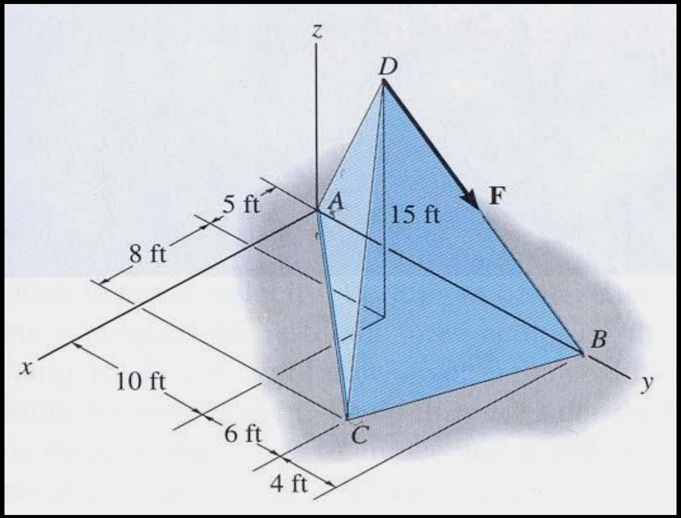
1) We need to use $M_{AC} = \boldsymbol{u_{AC}} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F_{DB}})$

2) Find $\boldsymbol{u_{AC}} = \boldsymbol{r_{AC}} / r_{AC}$

3) Find $\boldsymbol{F_{DB}} = 80 \text{ lb } \boldsymbol{u_{DB}} = 80 \text{ lb } (\boldsymbol{r_{DB}} / r_{DB})$

4) Complete the triple scalar product.

SOLUTION



$\mathbf{r}_{AB} = \{ 20\,\mathbf{j} \}$ ft

$\mathbf{r}_{AC} = \{ 13\,\mathbf{i} + 16\,\mathbf{j} \}$ ft

$\mathbf{r}_{DB} = \{ -5\,\mathbf{i} + 10\,\mathbf{j} - 15\,\mathbf{k} \}$ ft

$$\mathbf{u}_{AC} = (13\,\mathbf{i} + 16\,\mathbf{j})\text{ ft} / (13^2 + 16^2)^{½}\text{ ft}$$

$$= 0.6306\,\mathbf{i} + 0.7761\,\mathbf{j}$$

$$\mathbf{F}_{DB} = 80\,\{\mathbf{r}_{DB} / (5^2 + 10^2 + 15^2)^{½}\}\text{ lb}$$

$$= \{-21.38\,\mathbf{i} + 42.76\,\mathbf{j} - 64.14\,\mathbf{k}\}\text{ lb}$$

Solution (continued)

Now find the triple product, $M_{AC} = \boldsymbol{u}_{AC} \bullet (\boldsymbol{r}_{AB} \times \boldsymbol{F}_{DB})$

$$M_{AC} = \begin{vmatrix} 0.6306 & 0.7706 & 0 \\ 0 & 20 & 0 \\ -21.38 & 42.76 & -64.14 \end{vmatrix} \begin{matrix} \\ \text{ft} \\ \text{lb} \end{matrix}$$

$M_{AC} = 0.6306 \{20 (-64.14) - 0 - 0.7706 (0 - 0)\}$ lb·ft

$= -809$ lb·ft

The negative sign indicates that the sense of $\boldsymbol{M}_{AC}$ is opposite to that of $\boldsymbol{u}_{AC}$

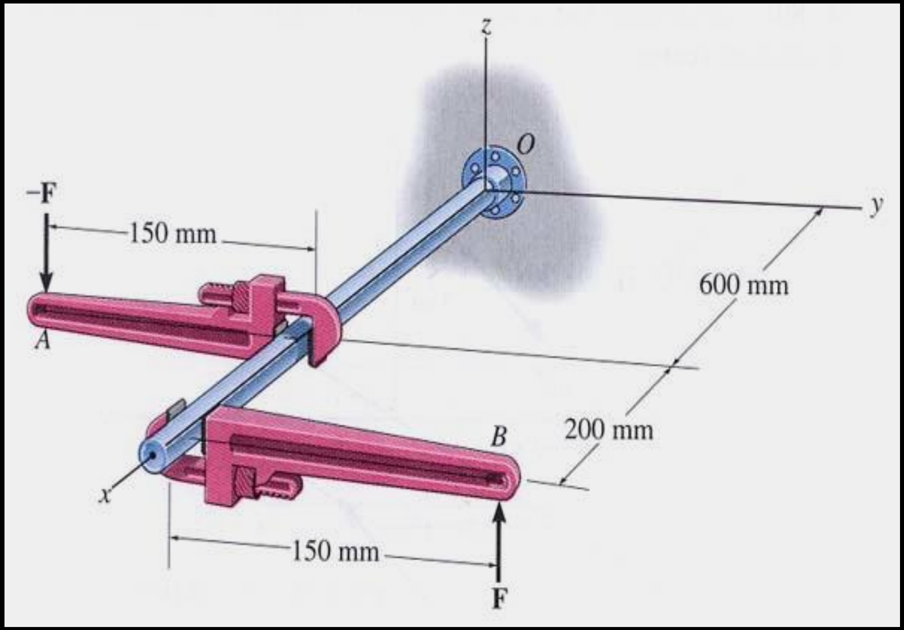
MOMENT OF A COUPLE

Today's Objectives:

Students will be able to

a) define a couple, and,

b) determine the moment of a couple.



In-Class activities:

- Check homework, if any
- Reading quiz
- Applications
- Moment of a Couple
- Concept quiz
- Group problem solving
- Attention quiz

READING QUIZ

1. In statics, a couple is defined as __________ separated by a perpendicular distance.

   A) two forces in the same direction.

   B) two forces of equal magnitude.

   C) two forces of equal magnitude acting in the same direction.

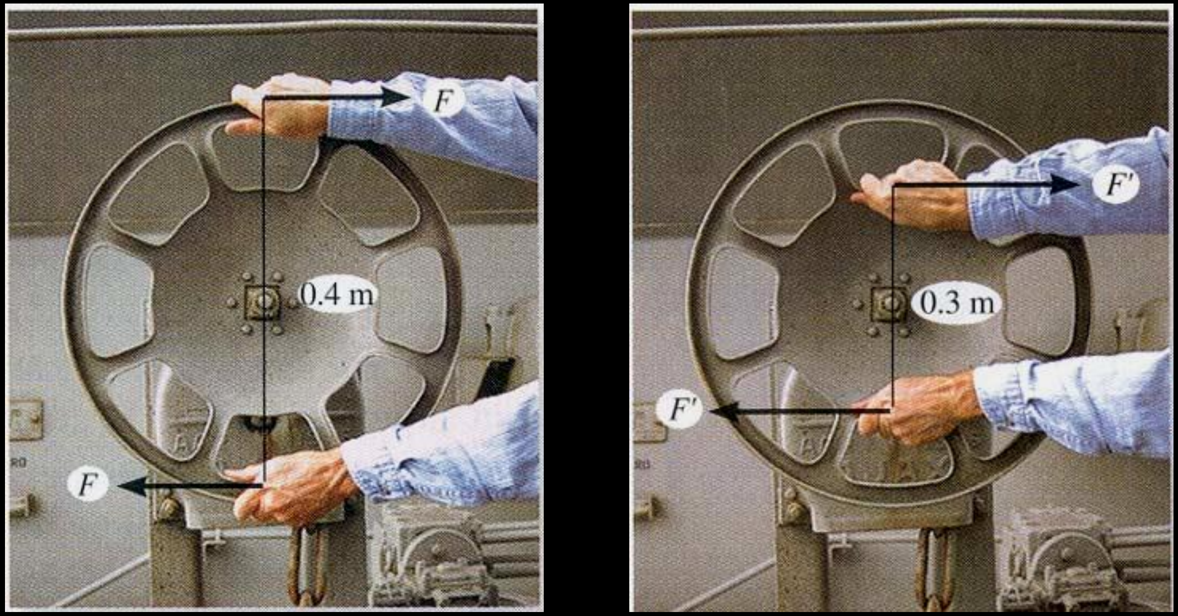
   D) two forces of equal magnitude acting in opposite directions.

2. The moment of a couple is called a _________ vector.

   A) free B) spin

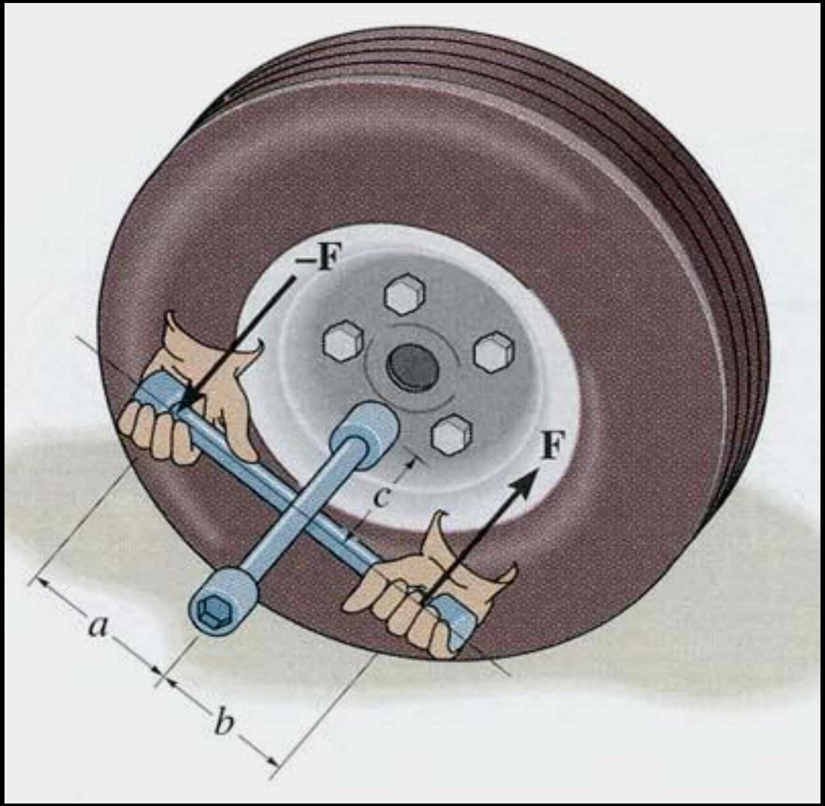
   C) romantic D) sliding

APPLICATIONS



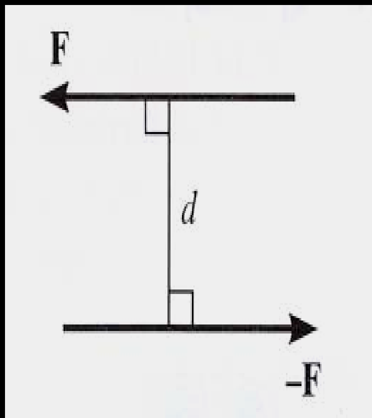
A torque or moment of 12 N · m is required to rotate the wheel. Which one of the two grips of the wheel above will require less force to rotate the wheel?

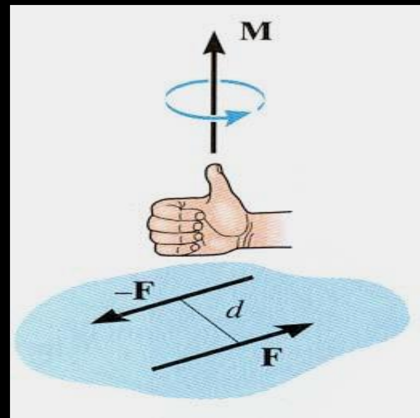
APPLICATIONS (continued)



The crossbar lug wrench is being used to loosen a lug net. What is the effect of changing dimensions a, b, or c on the force that must be applied?

MOMENT OF A COUPLE





A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

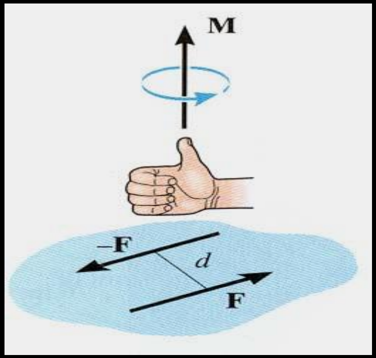
The moment of a couple is defined as

M_O = F d (using a scalar analysis) or as

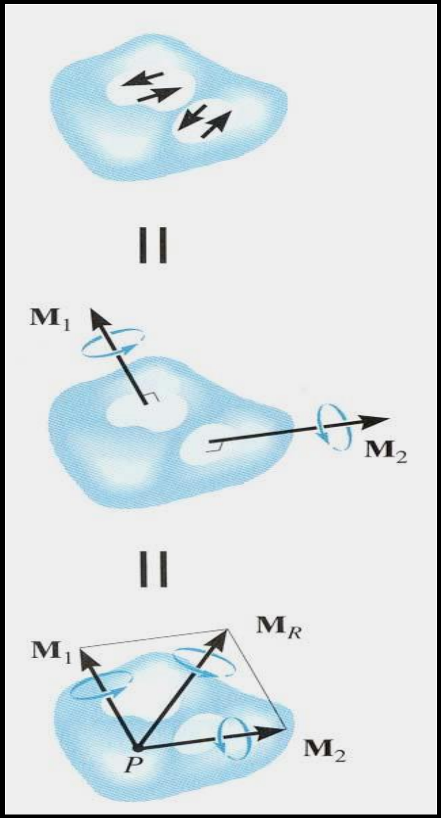
$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).

Here ***r*** is any position vector from the line of action of ***–F*** to the line of action of ***F***.

MOMENT OF A COUPLE (continued)



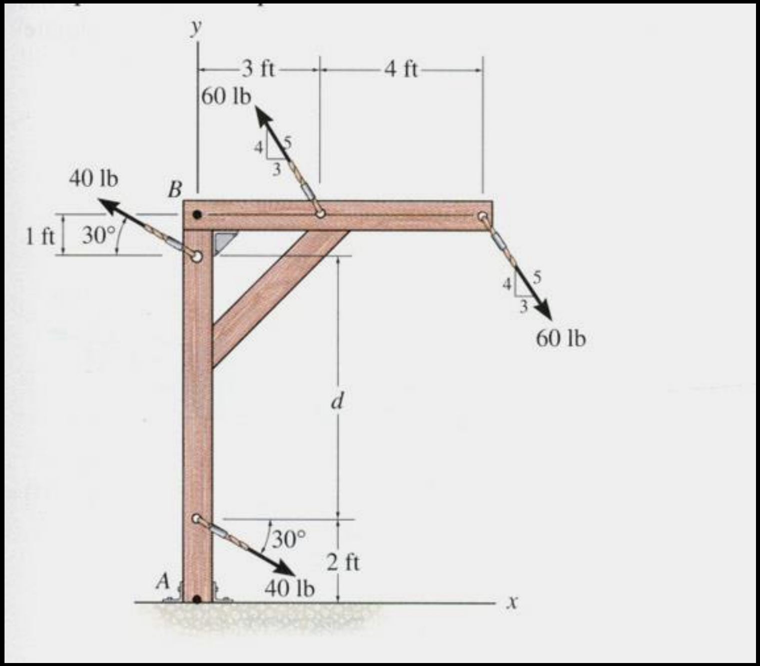
The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F d



Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a **free vector**. It can be moved anywhere on the body and have the same external effect on the body.

Moments due to couples can be added using the same rules as adding any vectors.

EXAMPLE - SCALAR APPROACH



Given: Two couples act on the beam and d equals 8 ft.

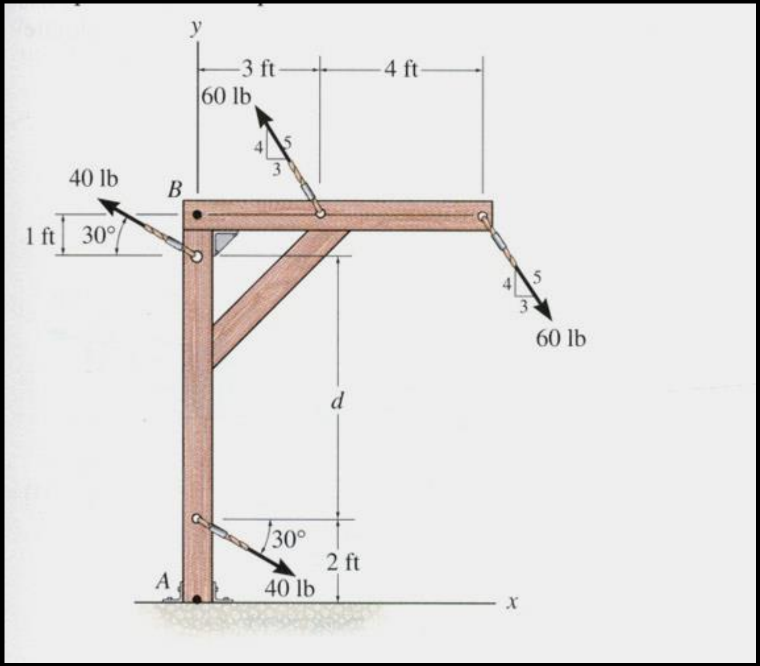
Find: The resultant couple

Plan:

1) Resolve the forces in x and y directions so they can be treated as couples.

2) Determine the net moment due to the two couples.

Solution:



The x and y components of the top 60 lb force are:

(4/5)(60 lb) = 48 lb vertically up

(3/5)(60 lb) = 36 lb to the left

Similarly for the top 40 lb force:

(40 lb) (sin 30°) up

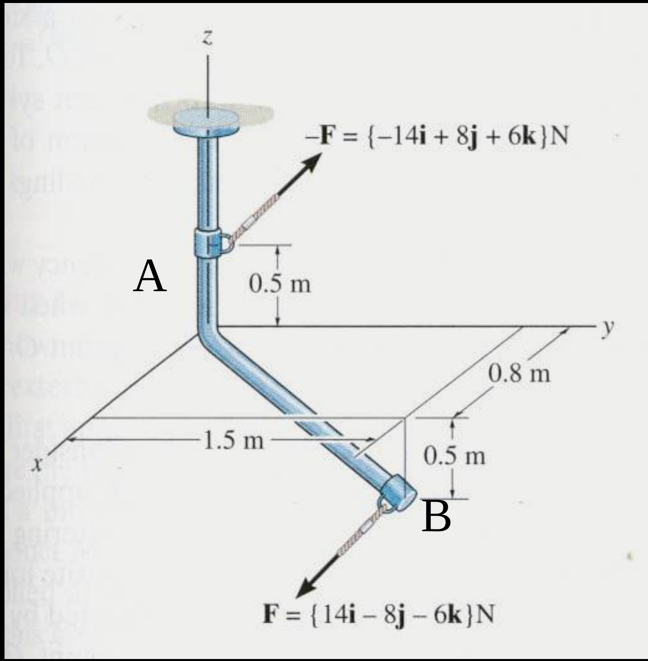
(40 lb) (cos 30°) to the left

The net moment equals to

$$+\circlearrowright \Sigma M = -(48\text{ lb})(4\text{ ft}) + (40\text{ lb})(\cos 30^\circ)(8\text{ft})$$

$$= -192.0 + 277.1 = 85.1\text{ ft}\cdot\text{lb}$$

EXAMPLE - VECTOR



Given: A force couple acting on the rod.

Find: The couple moment acting on the rod in Cartesian vector notation.

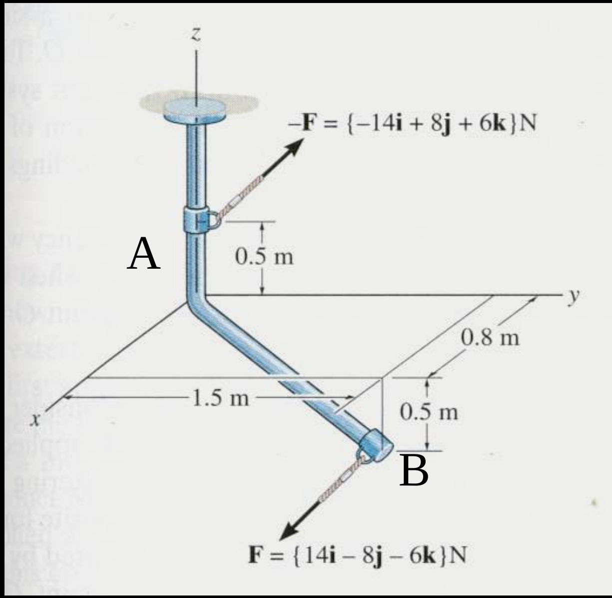
Plan:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

Solution:



$r_{AB} = \{0.8\,\boldsymbol{i} + 1.5\,\boldsymbol{j} - 1\,\boldsymbol{k}\}$ m

$F = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N

$M = r_{AB} \times F$

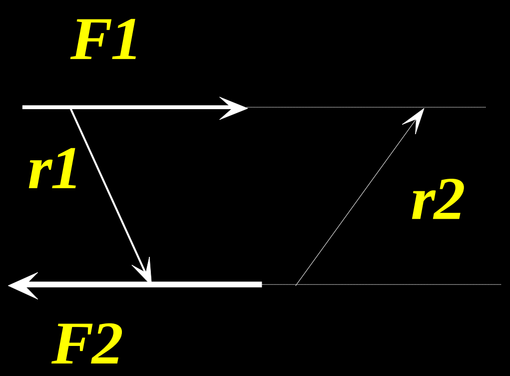
$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.8 & 1.5 & -1 \\ 14 & -8 & -6 \end{vmatrix} \text{ N·m}$$

$= \{\boldsymbol{i}\,(-9 - (-8)) - \boldsymbol{j}\,(-4.8 - (-14)) + \boldsymbol{k}\,(-4.8 - -14(1.5))\}$ N·m

$= \{-17\,\boldsymbol{i} - 9.2\,\boldsymbol{j} - 21\,\boldsymbol{k}\}$ N·m

CONCEPT QUIZ

1. F_1 and F_2 form a couple. The moment of the couple is given by ____ .



A) $r_1 \times F_1$ B) $r_2 \times F_1$

C) $F_2 \times r_1$ D) $r_2 \times F_2$

2. If three couples act on a body, the overall result is that

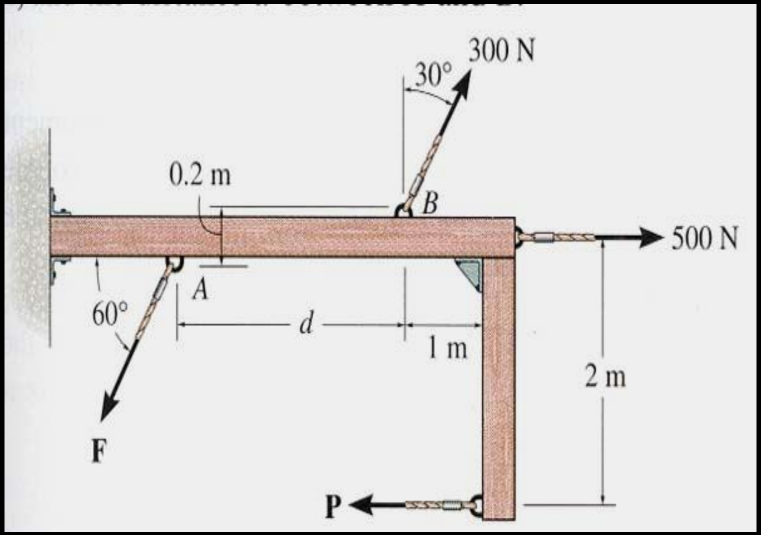
A) the net force is not equal to 0.

B) the net force and net moment are equal to 0.

C) the net moment equals 0 but the net force is not necessarily equal to 0.

D) the net force equals 0 but the net moment is not necessarily equal to 0 .

GROUP PROBLEM SOLVING - SCALAR



Given: Two couples act on the beam. The resultant couple is zero.

Find: The magnitudes of the forces P and F and the distance d.

<u>PLAN</u>:

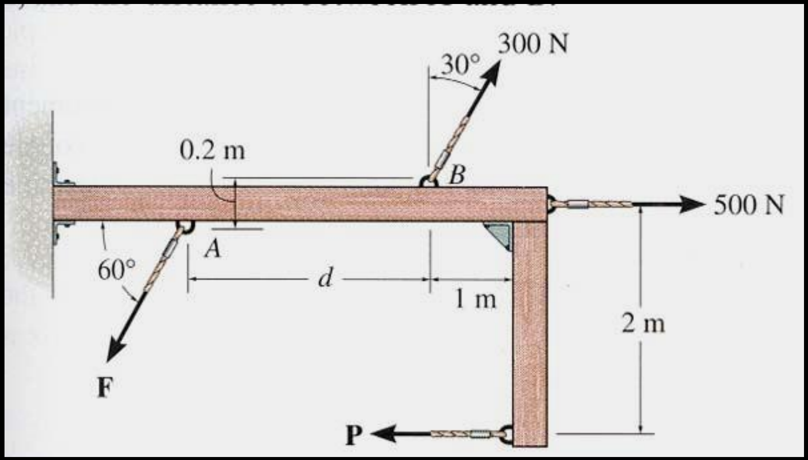
1) Use definition of a couple to find P and F.

2) Resolve the 300 N force in x and y directions.

3) Determine the net moment.

4) Equate the net moment to zero to find d.

Solution:



From the definition of a couple

P = 500 N and

F = 300 N.

Resolve the 300 N force into vertical and horizontal components. The vertical component is (300 cos 30º) N and the horizontal component is (300 sin 30º) N.

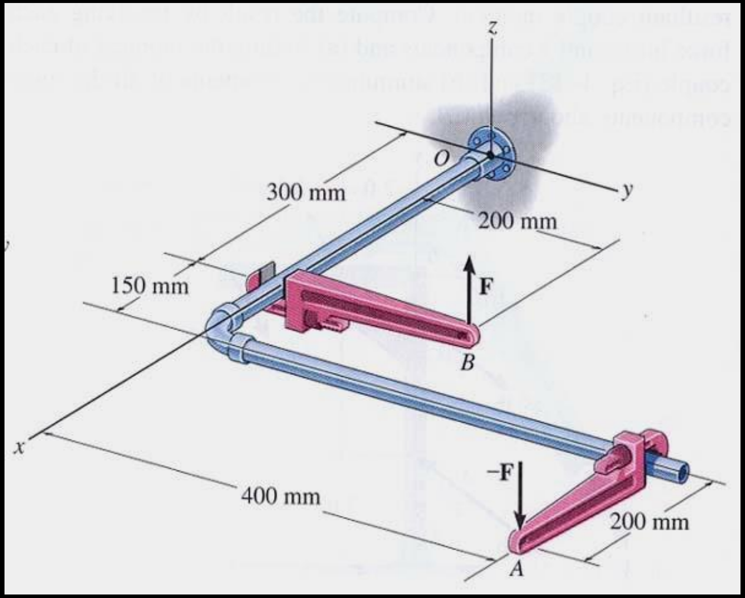
It was given that the net moment equals zero. So

$$+\circlearrowright \ \Sigma M = -(500)(2) + (300 \cos 30^\circ)(d) + (300 \sin 30^\circ)(0.2) = 0$$

Now solve this equation for d.

$$d = (1000 - 60 \sin 30^\circ) / (300 \cos 30^\circ) = 3.96 \text{ m}$$

GROUP PROBLEM SOLVING - VECTOR



Given: $\boldsymbol{F} = \{25\ \boldsymbol{k}\}$ N and

$-\boldsymbol{F} = \{-\ 25\ \boldsymbol{k}\}$ N

Find: The couple moment acting on the pipe assembly using Cartesian vector notation.

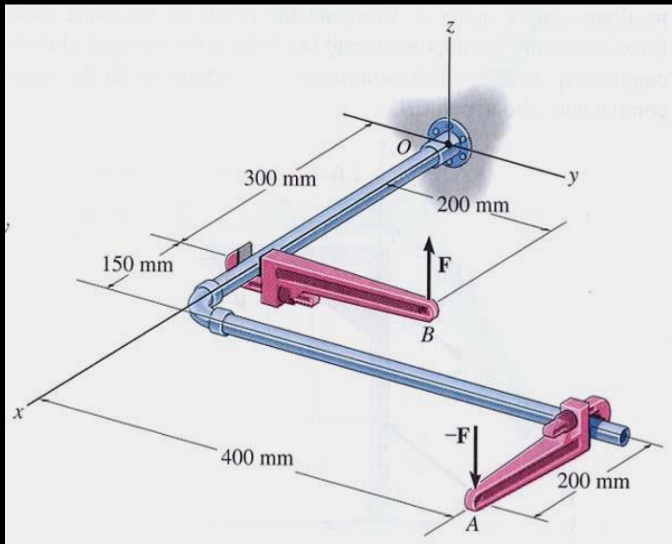
PLAN:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{25\ \boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

SOLUTION



$\mathbf{r}_{AB} = \{ -350\,\mathbf{i} - 200\,\mathbf{j} \}$ mm

$= \{ -0.35\,\mathbf{i} - 0.2\,\mathbf{j} \}$ m

$\mathbf{F} = \{25\,\mathbf{k}\}$ N

$\mathbf{M} = \mathbf{r}_{AB} \times \mathbf{F}$

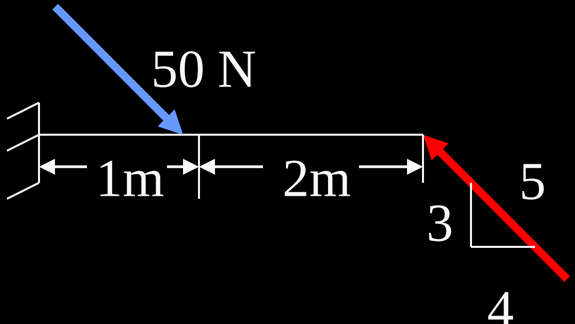
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix} \ \text{N} \cdot \text{m}$$

$= \{ \mathbf{i}\,(-5 - 0) - \mathbf{j}\,(-8.75 - 0) + \mathbf{k}\,(0) \}$ N · m

$= \{ -5\,\mathbf{i} + 8.75\,\mathbf{j} \}$ N · m

ATTENTION QUIZ

1. A couple is applied to the beam as shown. Its moment equals _____ N·m.

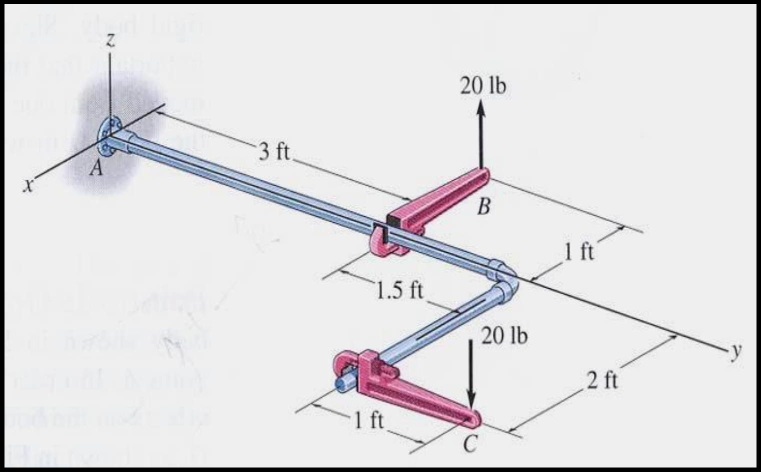


A) 50 B) 60

C) 80 D) 100

2. You can determine the couple moment as $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$

If $\boldsymbol{F}$ = { -20 $\boldsymbol{k}$} lb, then $\boldsymbol{r}$ is



A) $\boldsymbol{r}_{BC}$ B) $\boldsymbol{r}_{AB}$

C) $\boldsymbol{r}_{CB}$ D) $\boldsymbol{r}_{AC}$

End of the Lecture

Let Learning Continue