Learning unit 3

EQUILIBRIUM OF A RIGID BODY

EQUILIBRIUM OF A RIGID BODY

Today's Objectives:

Students will be able to

a) Identify support reactions, and,

b) Draw a free diagram.

In-Class Activities:

- Check homework, if any
- Reading Quiz
- Applications
- Support reactions
- Free – body diagram
- Concept quiz
- Group problem solving
- Attention quiz

READING QUIZ

1. If a support prevents translation of a body, then the support exerts a ___________ on the body.

   A) couple moment

   B) force

   C) Both A and B.

   D) None of the above

2. Internal forces are _________ shown on the free body diagram of a whole body.

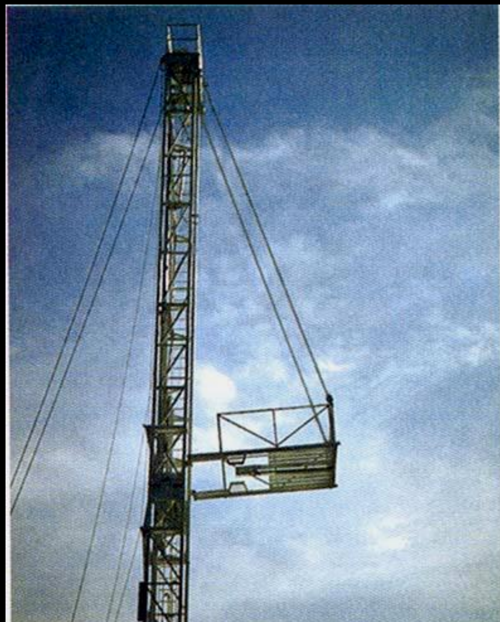
   A) always

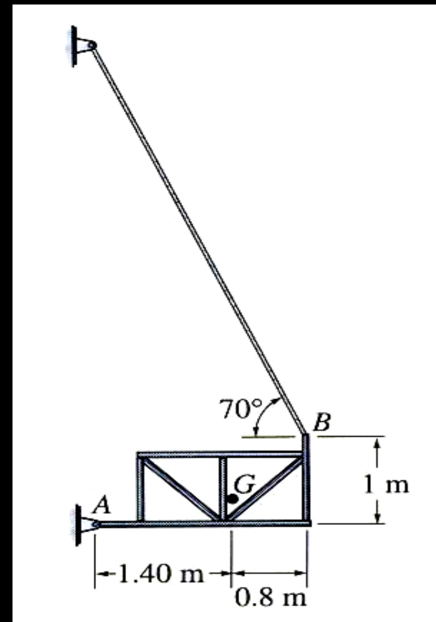
   B) often

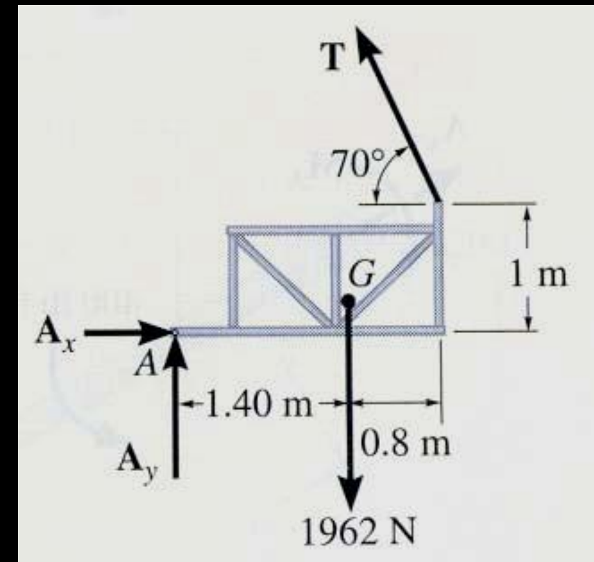
   C) rarely

   D) never

APPLICATIONS



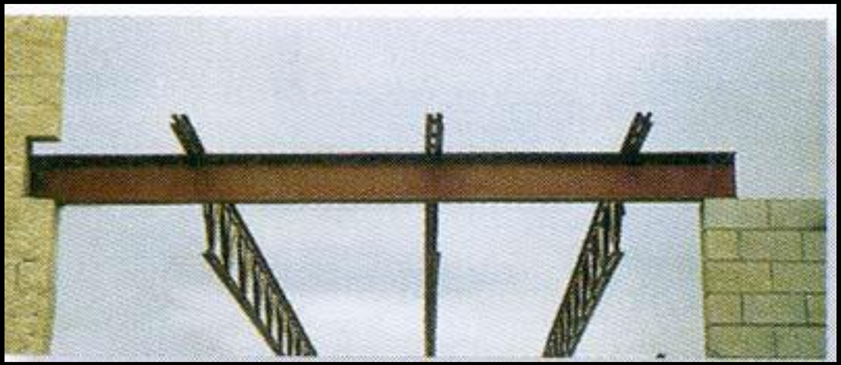


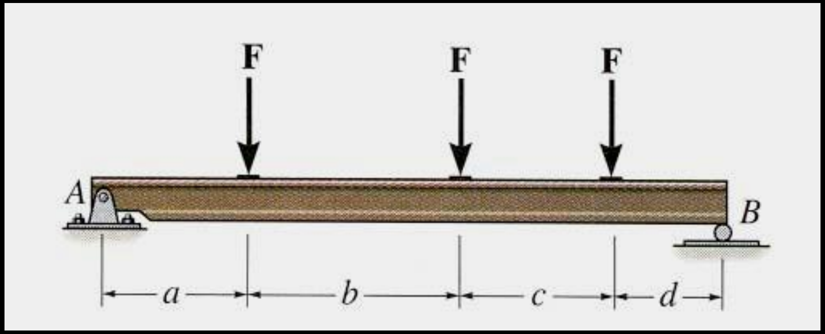


A 200 kg platform is suspended off an oil rig. How do we determine the force reactions at the joints and the forces in the cables?

How are the idealized model and the free body diagram used to do this? Which diagram above is the idealized model?

APPLICATIONS (continued)

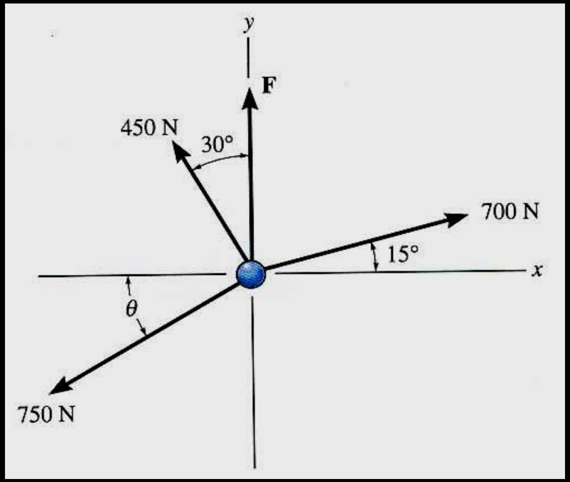




A steel beam is used to support roof joists. How can we determine the support reactions at A & B?

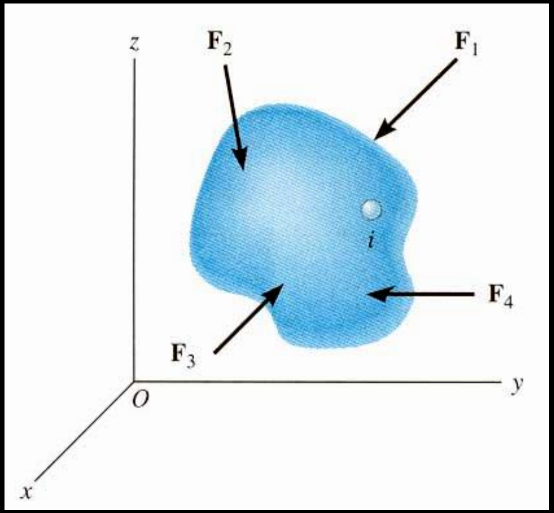
Again, how can we make use of an idealized model and a free body diagram to answer this question?

CONDITIONS FOR RIGID-BODY EQUILIBRIUM



Forces on a particle

In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body (due to the moments created by the forces).

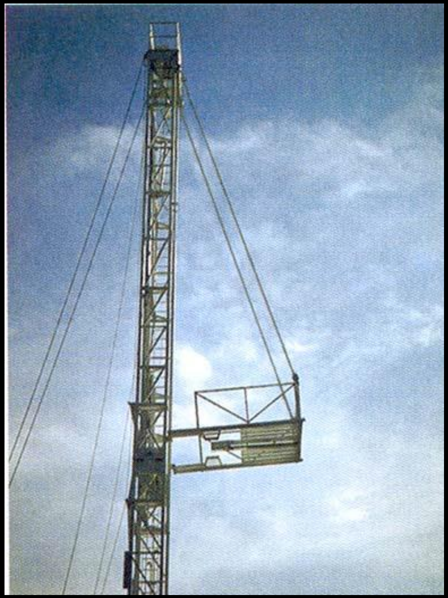


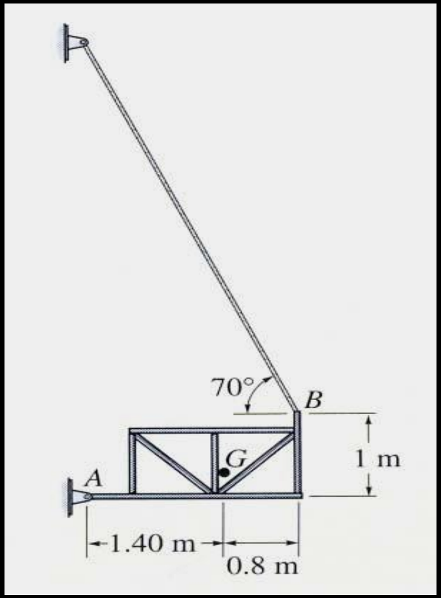
Forces on a rigid body

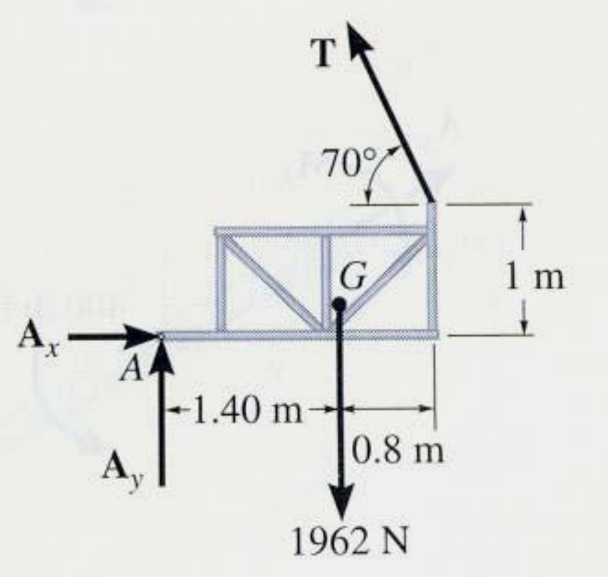
For a rigid body to be in equilibrium, the net force as well as the net moment about any arbitrary point O must be equal to zero.

$\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$

THE PROCESS OF SOLVING RIGID BODY EQUILIBRIUM PROBLEMS





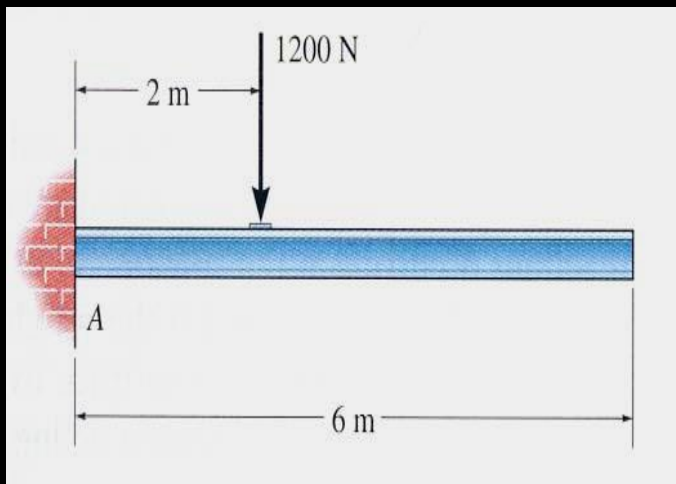


For analyzing an actual physical system, first we need to create an idealized model.

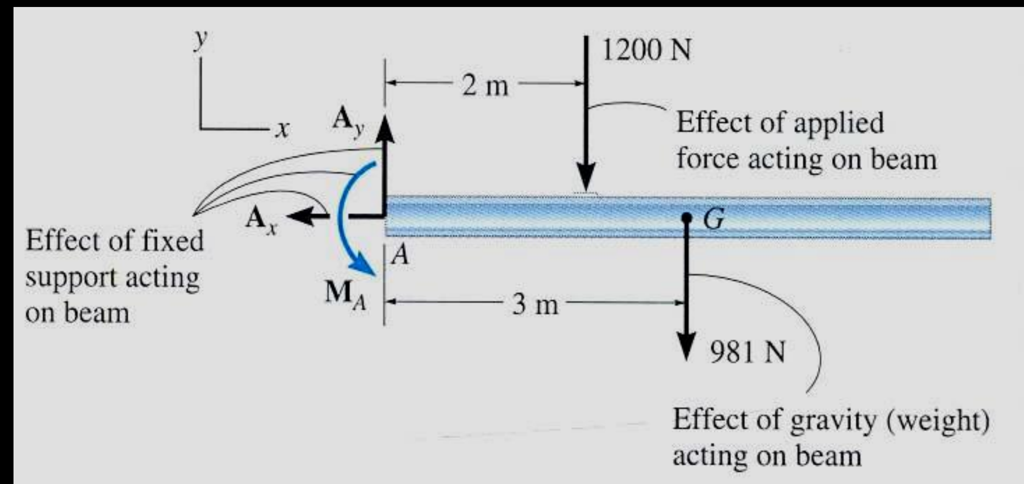
Then we need to draw a free-body diagram showing all the external (active and reactive) forces.

Finally, we need to apply the equations of equilibrium to solve for any unknowns.

PROCEDURE FOR DRAWING A FREE BODY DIAGRAM



Idealized model

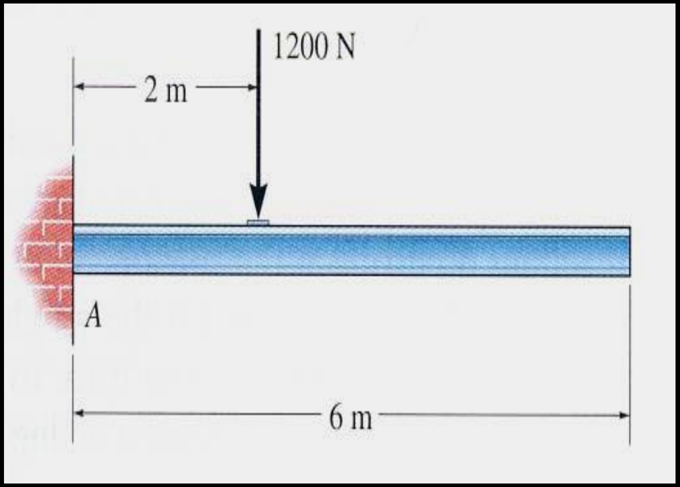


Free body diagram

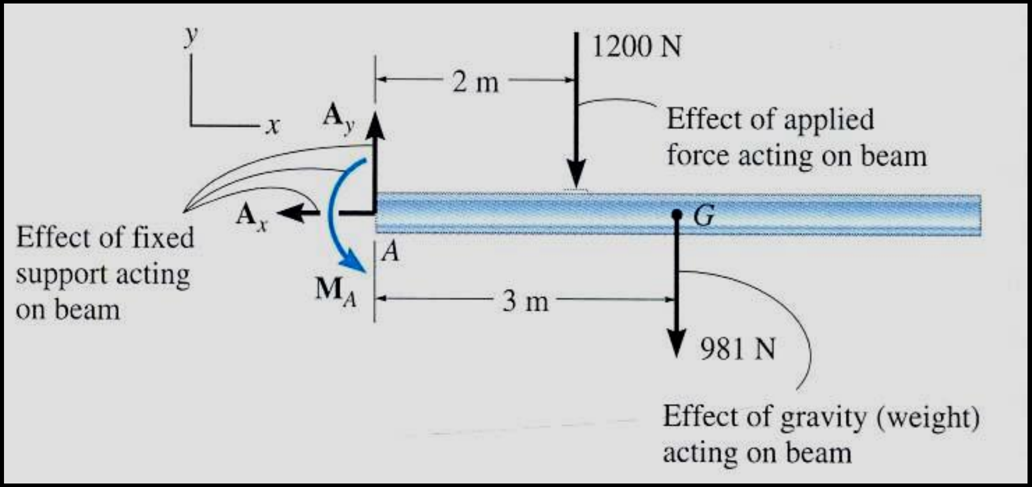
1. Draw an outlined shape. Imagine the body to be isolated or cut "free" from its constraints and draw its outlined shape.

2. Show all the external forces and couple moments. These typically include: a) applied loads, b) support reactions, and, c) the weight of the body.

PROCEDURE FOR DRAWING A FREE BODY DIAGRAM (continued)



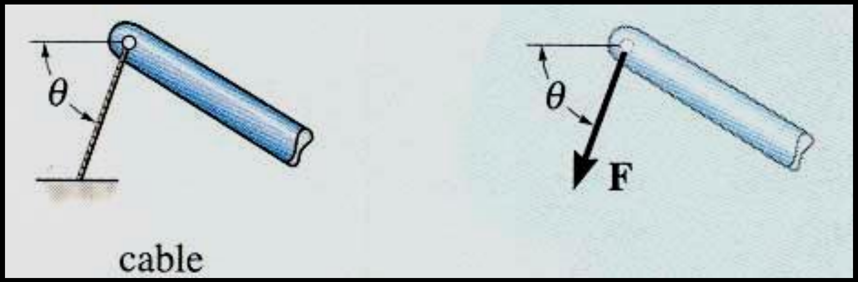
Idealized model

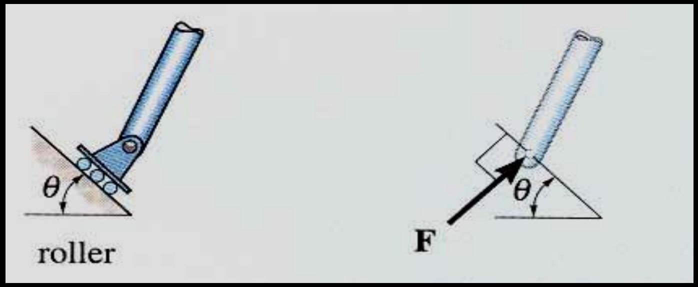


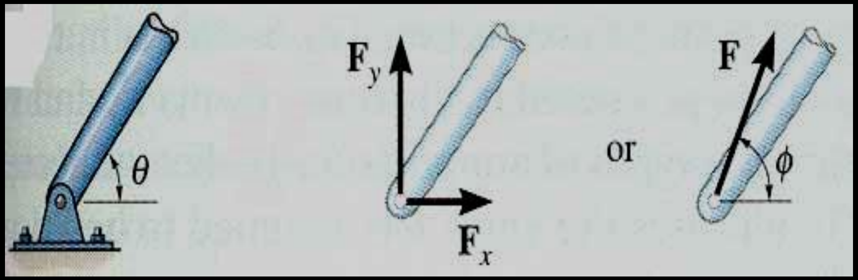
Free body diagram

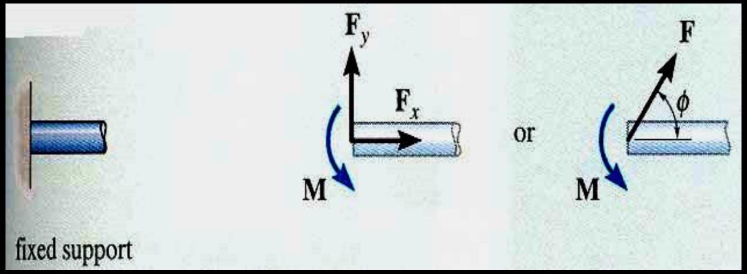
3. Label loads and dimensions: All known forces and couple moments should be labeled with their magnitudes and directions. For the unknown forces and couple moments, use letters like A_x, A_y, M_A, etc.. Indicate any necessary dimensions.

SUPPORT REACTIONS IN 2-D





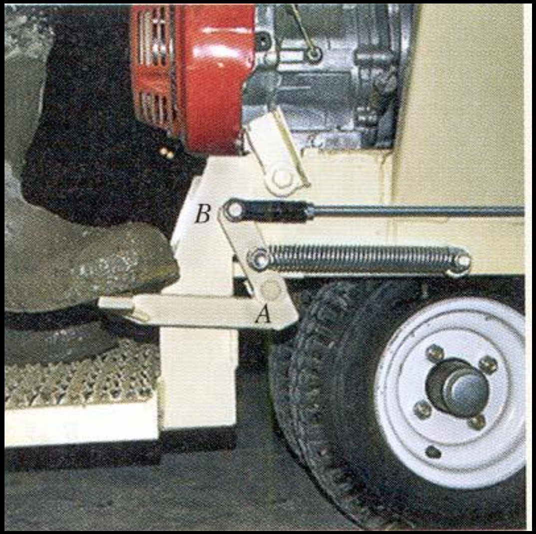




A few examples are shown above. Other support reactions are given in your textbook (in Table 5-1).

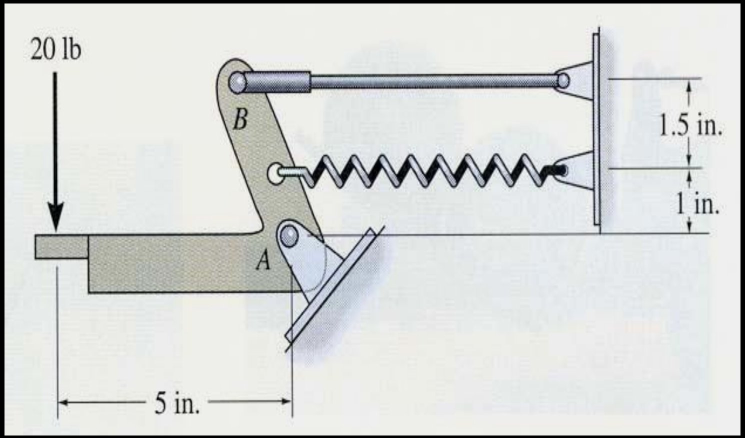
As a general rule, if a support prevents translation of a body in a given direction, then a force is developed on the body in the opposite direction. Similarly, if rotation is prevented, a couple moment is exerted on the body.

EXAMPLE

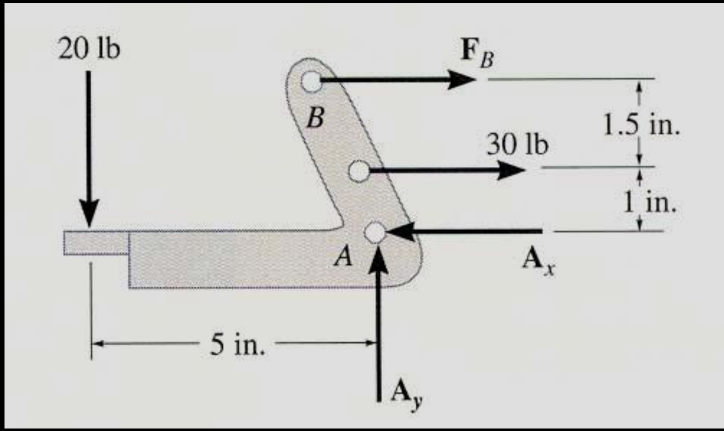


Given: An operator applies 20 lb to the foot pedal. A spring with k = 20 lb/in is stretched 1.5 in.

Draw: A free body diagram of the foot pedal.

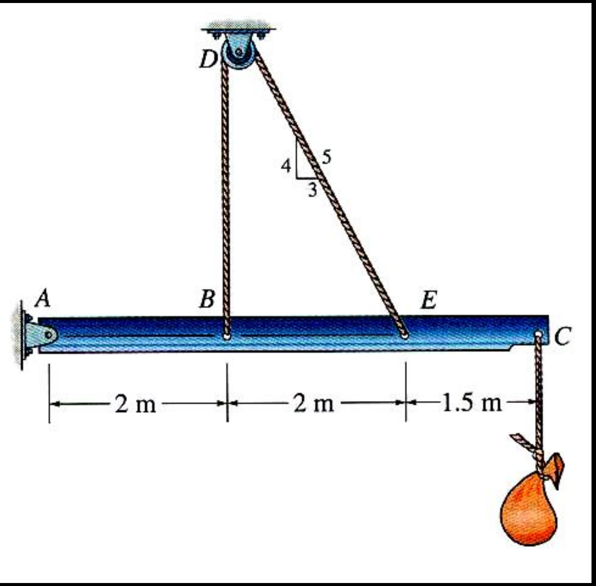


The idealized model



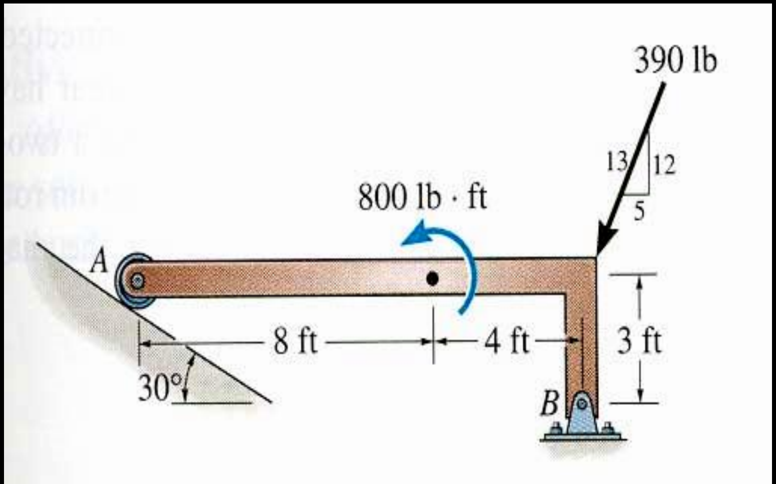
The free body diagram

CONCEPT QUIZ



1. The beam and the cable (with a frictionless pulley at D) support an 80 kg load at C. In a FBD of only the beam, there are how many unknowns?
   A) 2 forces and 1 couple moment
   B) 3 forces and 1 couple moment
   C) 3 forces
   D) 4 forces

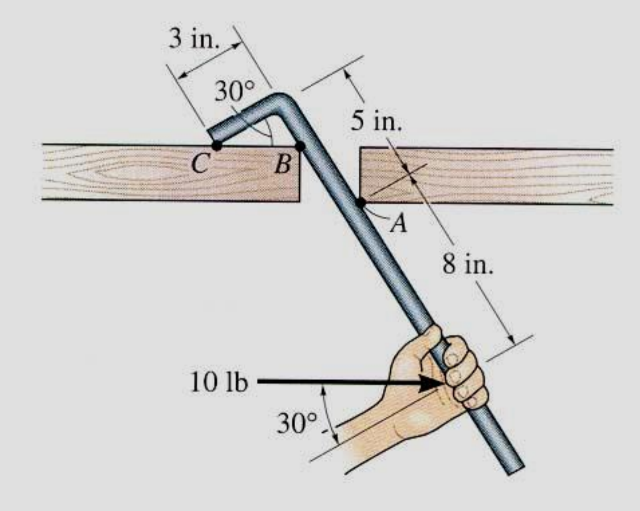
CONCEPT QUIZ



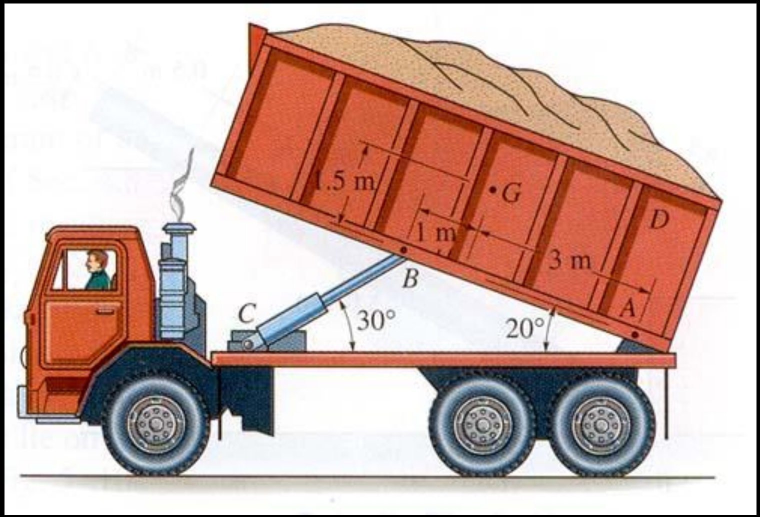
2. If the directions of the force and the couple moments are reversed, then what will happen to the beam?

   A) The beam will lift from A.
   B) The beam will lift at B.
   C) The beam will be restrained.
   D) The beam will break.

GROUP PROBLEM SOLVING



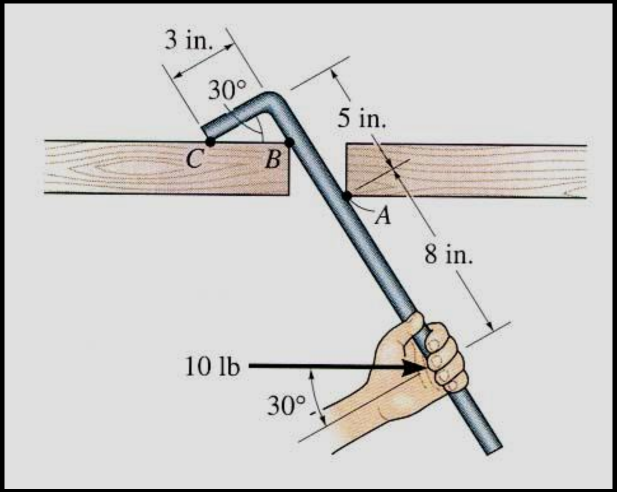
Problem 5-9



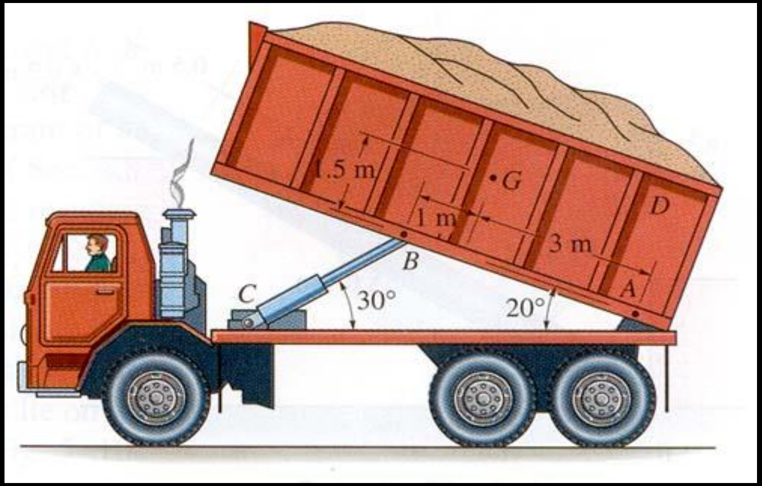
Draw a FBD of the bar, which has smooth points of contact at A, B, and C.

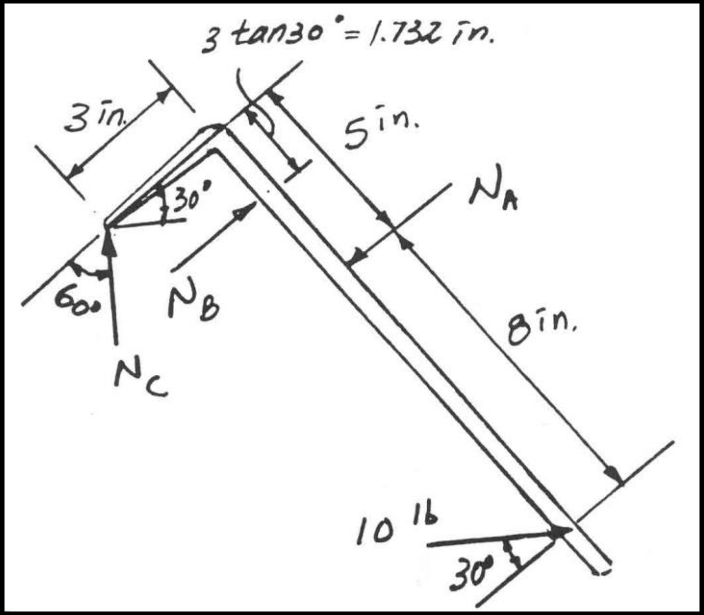
Draw a FBD of the 5000 lb dumpster (D). It is supported by a pin at A and the hydraulic cylinder BC (treat as a short link).

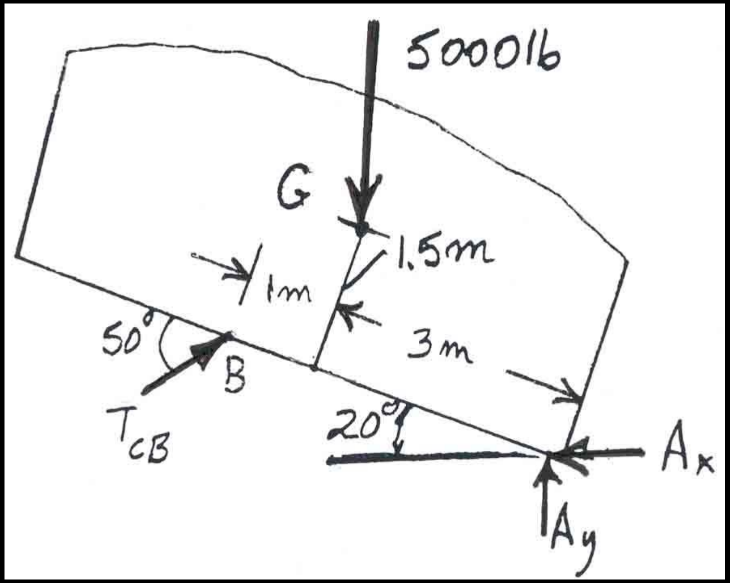
GROUP PROBLEM SOLVING (continued)



Problem 5-9







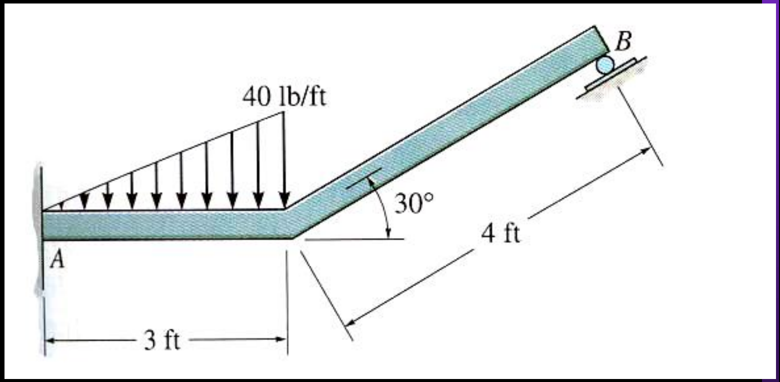
ATTENTION QUIZ

1. Internal forces are not shown on a free body diagram because the internal forces are______. (Choose the most appropriate answer.)

   A) equal to zero
   B) equal and opposite and they do not affect the calculations
   C) negligibly small
   D) not important

2. How many unknown support reactions are there in this problem?

   A) 2 forces and 2 couple moments
   B) 1 force and 2 couple moments
   C) 3 forces
   D) 3 forces and 1 couple moment



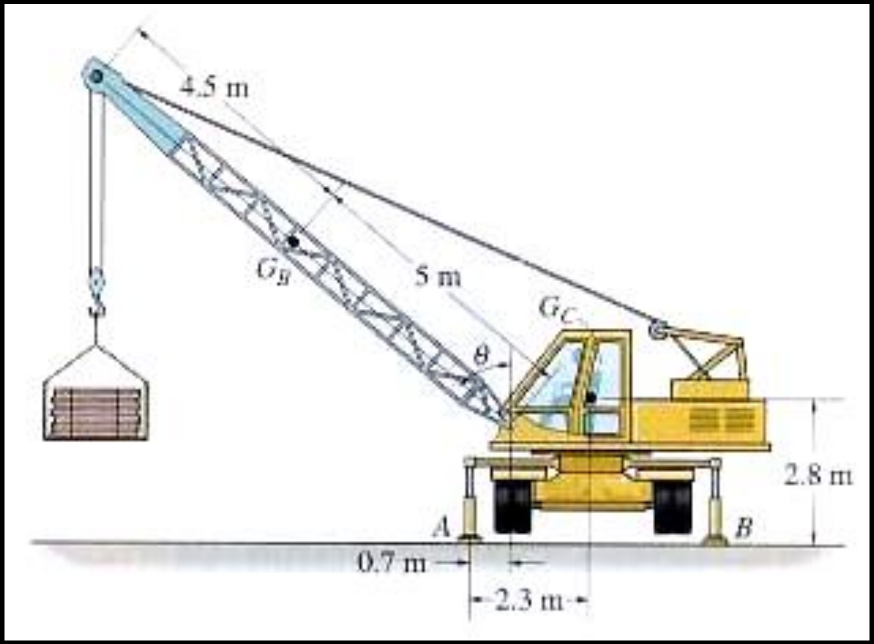
EQUATIONS OF EQUILIBRIUM IN 2-D

Today's Objectives:

Students will be able to

a) Apply equations of equilibrium to solve for unknowns, and,

b) Recognize two-force members.



In-Class Activities:

- Check homework, if any
- Reading quiz
- Applications
- Equations of equilibrium
- Two-force members
- Concept quiz
- Group problem solving
- Attention quiz

READING QUIZ

1. The three scalar equations $\sum F_X = \sum F_Y = \sum M_O = 0$, are ____ equations of equilibrium in two dimensions.

   A) incorrect

   B) the only correct

   C) the most commonly used

   D) not sufficient

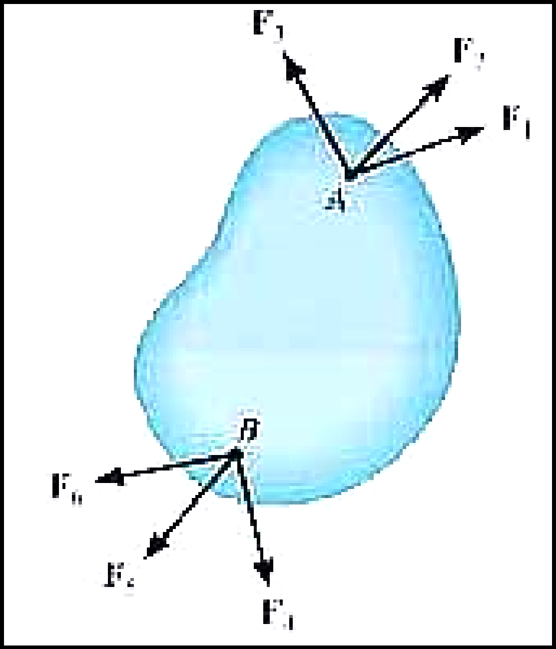
2. A rigid body is subjected to forces as shown. This body can be considered as a ______ member.

   A) single-force

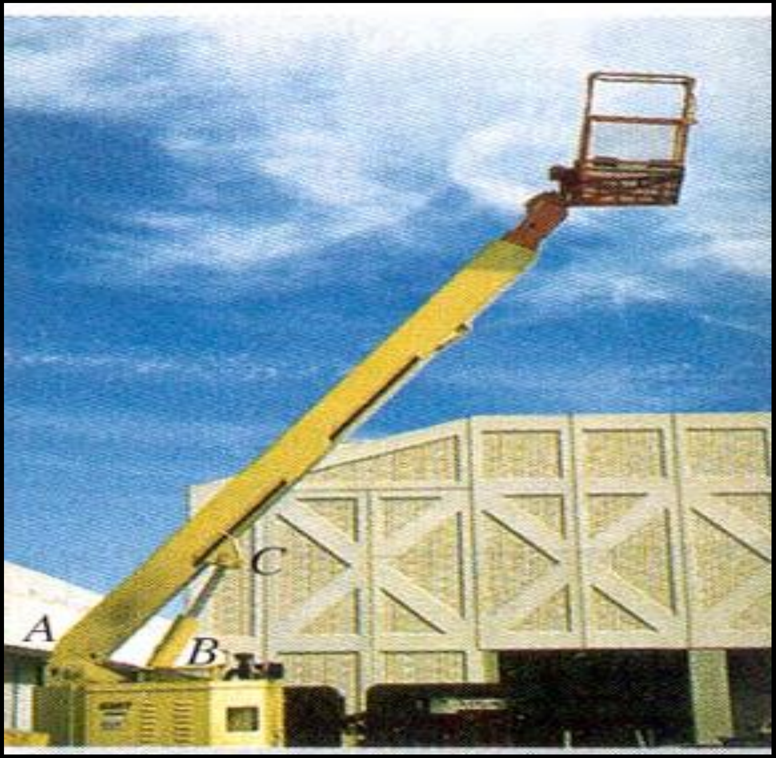
   B) two-force

   C) three-force

   D) six-force

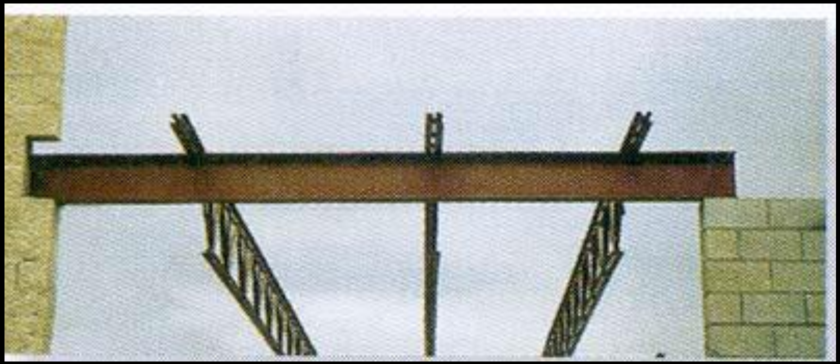


APPLICATIONS



For a given load on the platform, how can we determine the forces at the joint A and the force in the link (cylinder) BC?

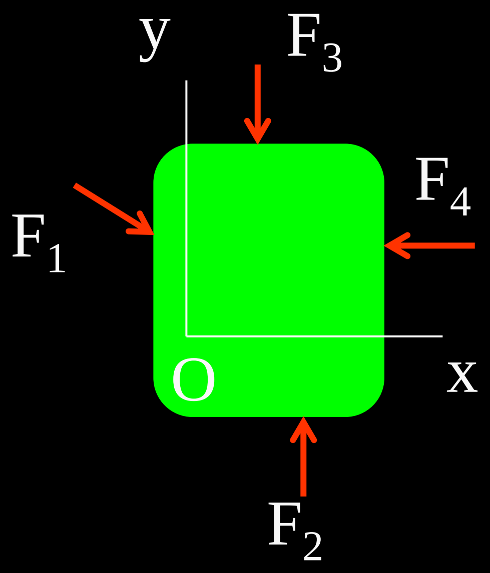
APPLICATIONS (continued)



A steel beam is used to support roof joists. How can we determine the support reactions at each end of the beam?

EQUATIONS OF EQUILIBRIUM

A body is subjected to a system of forces that lie in the x-y plane. When in equilibrium, the net force and net moment acting on the body are zero (as discussed earlier in Section 5.1). This 2-D condition can be represented by the three scalar equations:

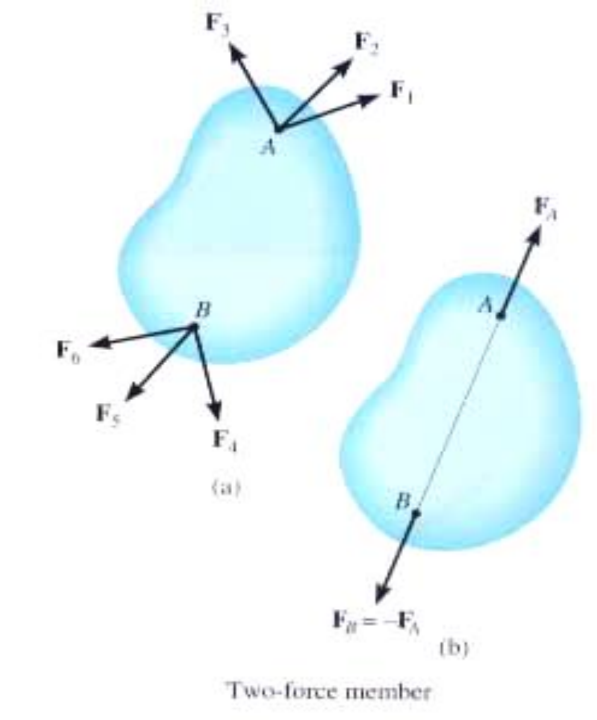


$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_O = 0$$

Where point O is any arbitrary point.

Please note that these equations are the ones most commonly used for solving 2-D equilibrium problems. There are two other sets of equilibrium equations that are rarely used. For your reference, they are described in the textbook.

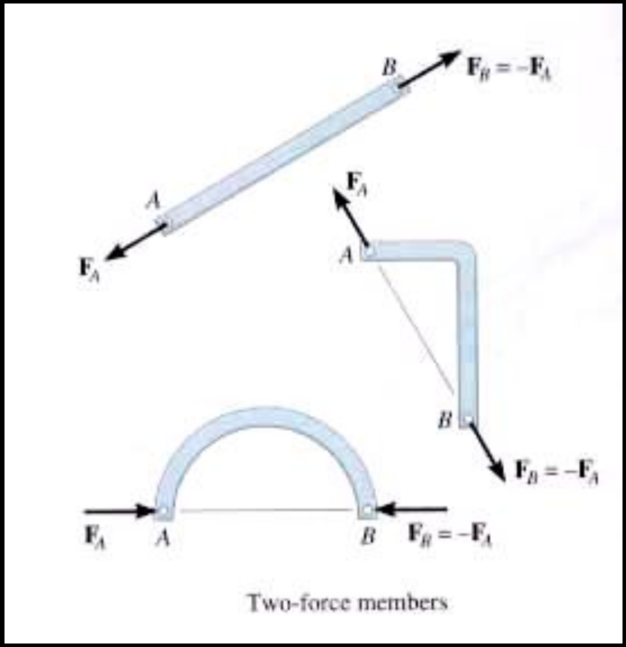
TWO-FORCE MEMBERS



The solution to some equilibrium problems can be simplified if we recognize members that are subjected to forces at only two points (e.g., at points A and B).

If we apply the equations of equilibrium to such a member, we can quickly determine that the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.

EXAMPLE OF TWO-FORCE MEMBERS



In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.

This fact simplifies the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).

STEPS FOR SOLVING 2-D EQUILIBRIUM PROBLEMS

1. If not given, establish a suitable x - y coordinate system.

2. Draw a free body diagram (FBD) of the object under analysis.

3. Apply the three equations of equilibrium (EofE) to solve for the unknowns.

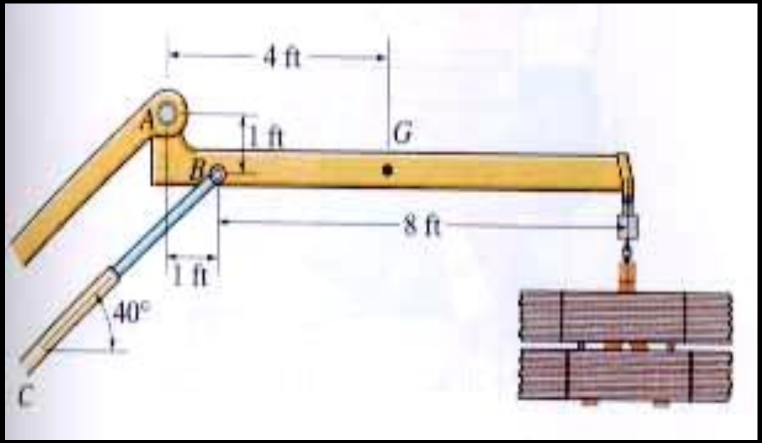
IMPORTANT NOTES

1. If we have more unknowns than the number of independent equations, then we have a statically indeterminate situation. We cannot solve these problems using just statics.

2. The order in which we apply equations may affect the simplicity of the solution. For example, if we have two unknown vertical forces and one unknown horizontal force, then solving $\sum F_X = O$ first allows us to find the horizontal unknown quickly.

3. If the answer for an unknown comes out as negative number, then the sense (direction) of the unknown force is opposite to that assumed when starting the problem.

EXAMPLE



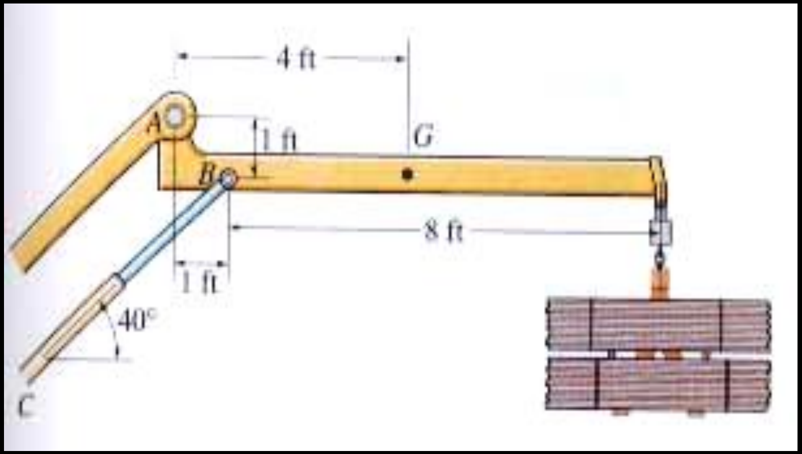
Given: Weight of the boom = 125 lb, the center of mass is at G, and the load = 600 lb.

Find: Support reactions at A and B.

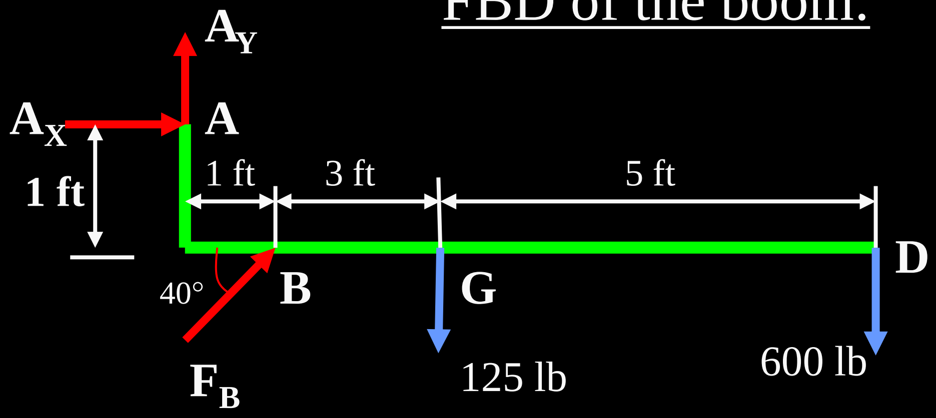
Plan:

1. Put the x and y axes in the horizontal and vertical directions, respectively.
2. Determine if there are any two-force members.
3. Draw a complete FBD of the boom.
4. Apply the EofE to solve for the unknowns.

EXAMPLE (Continued)



FBD of the boom:



Note: Upon recognizing CB as a two-force member, the number of unknowns at B are reduced from two to one. Now, using Eof E, we get,

$$\circlearrowleft + \sum M_A = 125 * 4 + 600 * 9 - F_B \sin 40° * 1 - F_B \cos 40° * 1 = 0$$

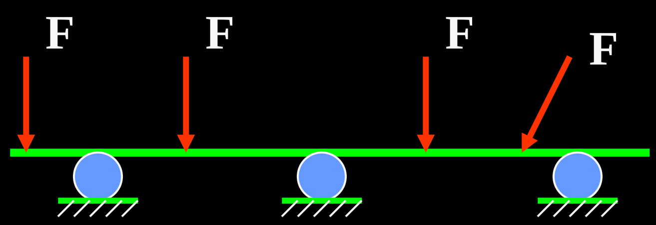
$F_B = 4188$ lb or 4190 lb

$$\rightarrow + \sum F_X = A_X + 4188 \cos 40° = 0; \quad A_X = -3210 \text{ lb}$$

$$\uparrow + \sum F_Y = A_Y + 4188 \sin 40° - 125 - 600 = 0; \quad A_Y = -1970 \text{ lb}$$

CONCEPT QUIZ

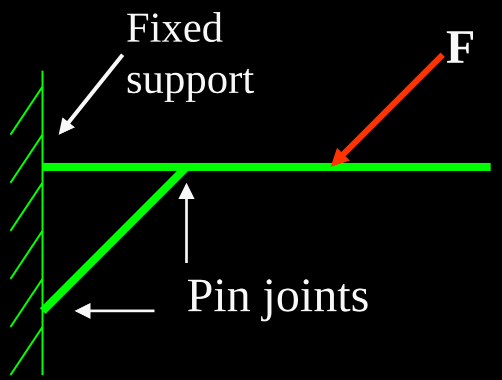
1. For this beam, how many support reactions are there and is the problem statically determinate?
   A) (2, Yes) B) (2, No)
   C) (3, Yes) D) (3, No)



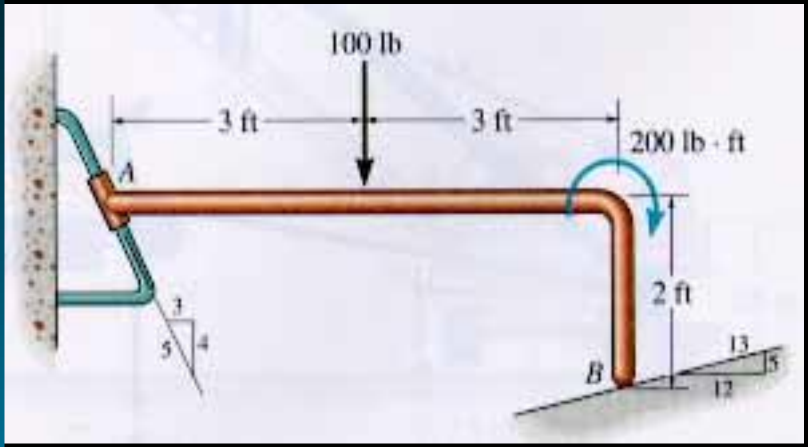
2. For the given beam loading: a) how many support reactions are there, b) is this problem statically determinate, and, c) is the structure stable?

   A) (4, Yes, No) B) (4, No, Yes)

   C) (5, Yes, No) D) (5, No, Yes)



GROUP PROBLEM SOLVING



Given: The load on the bent rod is supported by a smooth inclined surface at B and a collar at A. The collar is free to slide over the fixed inclined rod.

Find: Support reactions at A and B.

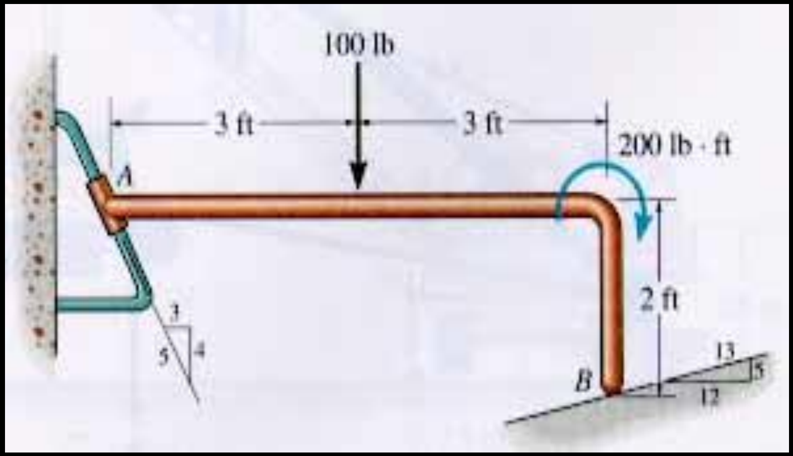
Plan:

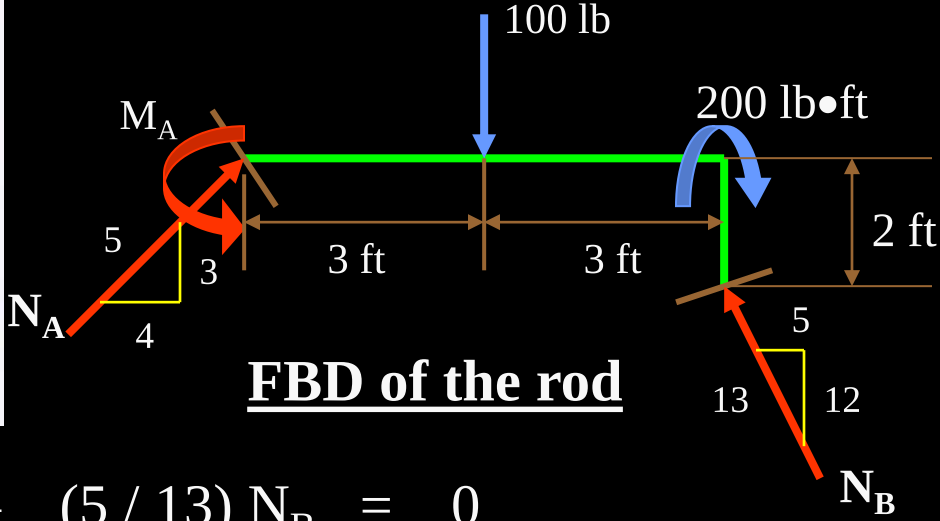
a) Establish the x – y axes.

b) Draw a complete FBD of the bent rod.

c) Apply the EofE to solve for the unknowns.

GROUP PROBLEM SOLVING (Continued)





FBD of the rod

$\rightarrow + \sum F_X = (4/5)\, N_A - (5/13)\, N_B = 0$

$\uparrow + \sum F_Y = (3/5)\, N_A - (12/13)\, N_B - 100 = 0$

Solving these two equations, we get

$N_B = 82.54$ or $\underline{82.5 \text{ lb}}$ and $N_A = 39.68$ or $\underline{39.7 \text{ lb}}$

$\circlearrowleft + \sum M_A = M_A - 100 * 3 - 200 + (12/13)\, N_B * 6 - (5/13)\, N_B * 2 = 0$

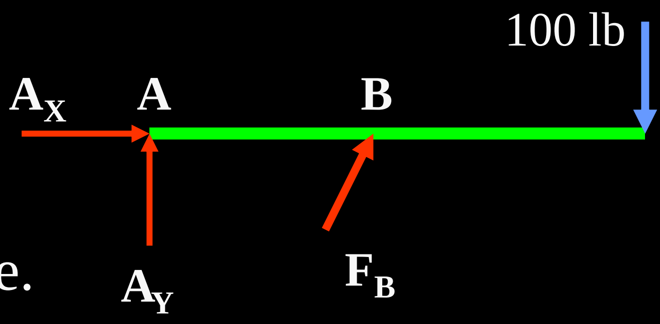
$\underline{M_A = 106 \text{ lb} \bullet \text{ft}}$

ATTENTION QUIZ

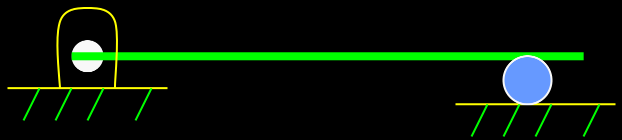
1. Which equation of equilibrium allows you to determine F_B right away?

   A) $\sum F_X = 0$ B) $\sum F_Y = 0$

   C) $\sum M_A = 0$ D) Any one of the above.



2. A beam is supported by a pin joint and a roller. How many support reactions are there and is the structure stable for all types of loadings?



   A) (3, Yes) B) (3, No)

   C) (4, Yes) D) (4, No)

RIGID BODY EQUILIBRIUM IN 3-D

Today's Objective:

Students will be able to

a) Identify support reactions in 3-D and draw a free body diagram, and,

b) apply the equations of equilibrium.

In-Class Activities:

- Check homework, if any
- Reading quiz
- Applications
- Support reactions in 3-D
- Equations of equilibrium
- Concept quiz
- Group problem solving
- Attention quiz

READING QUIZ

1. If a support prevents rotation of a body about an axis, then the support exerts a _________ on the body about that axis.

   A) couple moment B) force

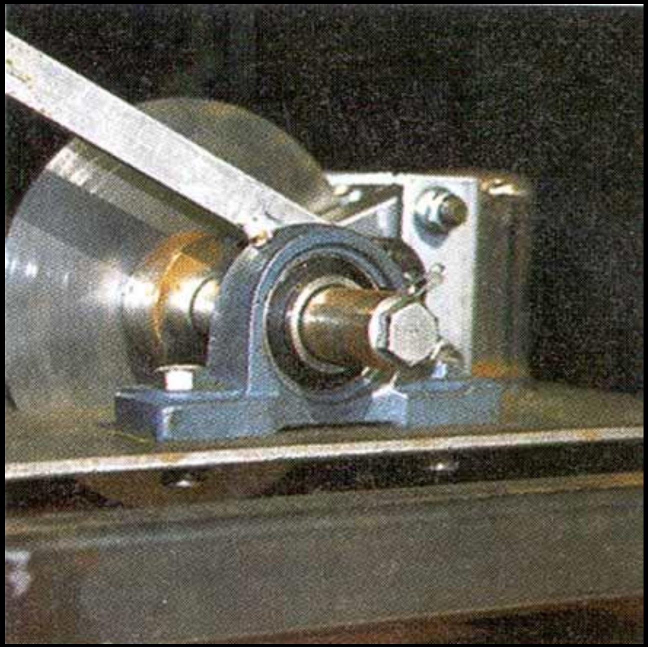
   C) Both A and B. D) None of the above.

2. When doing a 3-D problem analysis, you have _________ scalar equations of equilibrium.

   A) 2 B) 3 C) 4

   D) 5 E) 6

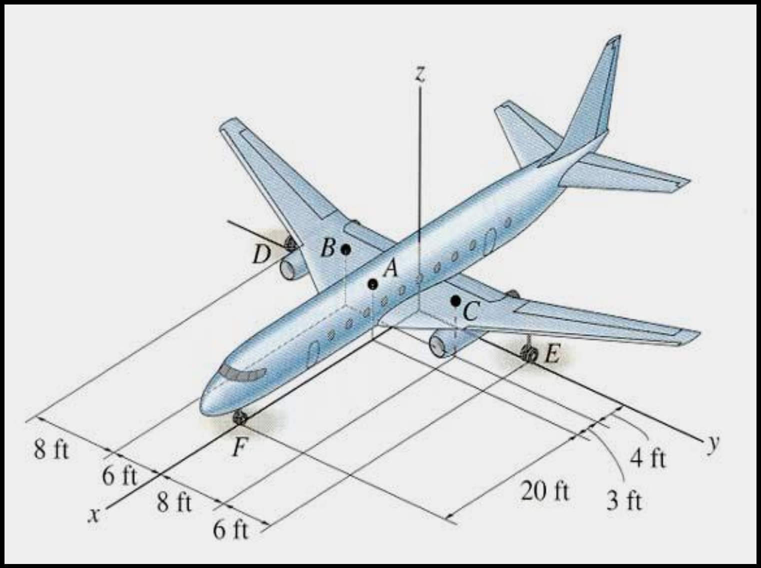
APPLICATIONS



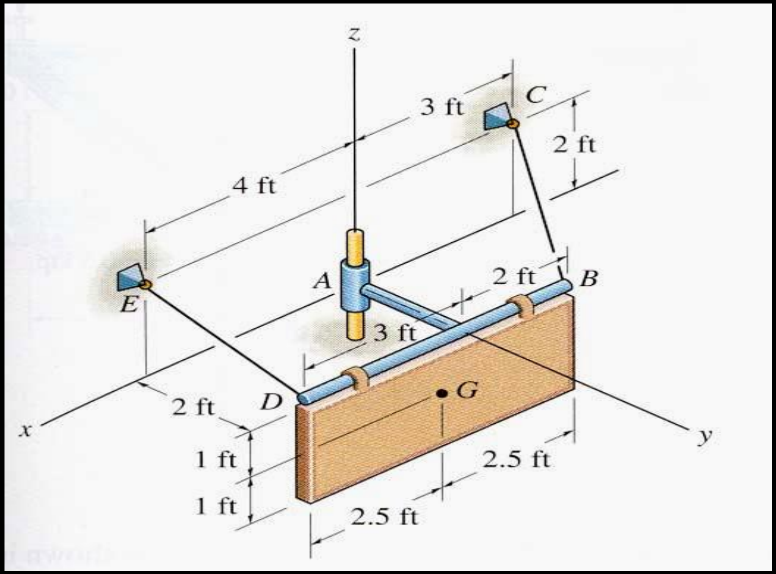
Ball-and-socket joints and journal bearings are often used in mechanical systems.

How can we determine the support reactions at these joints for a given loading?

APPLICATIONS (continued)

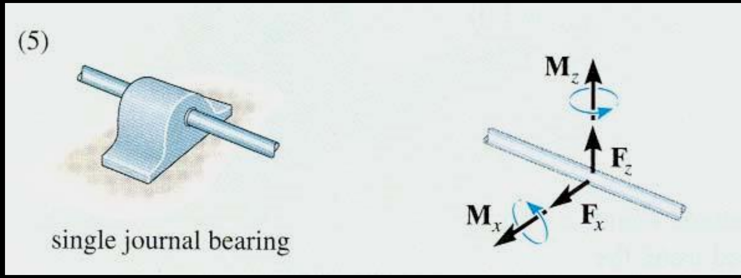


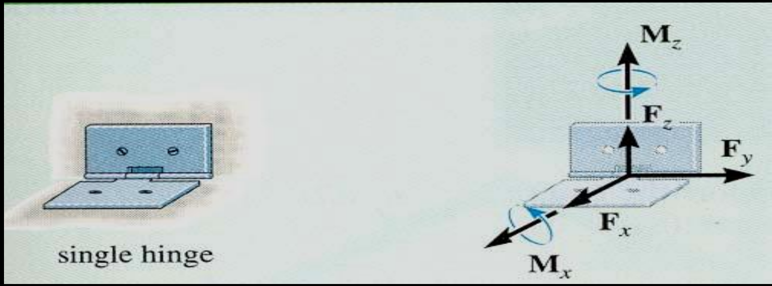
The weights of the fuselage and fuel act through A, B, and C. How will we determine the reactions at the wheels D, E and F ?



A 50 lb sign is kept in equilibrium using two cables and a smooth collar. How can we determine the reactions at these supports?

SUPPORT REACTIONS IN 3-D

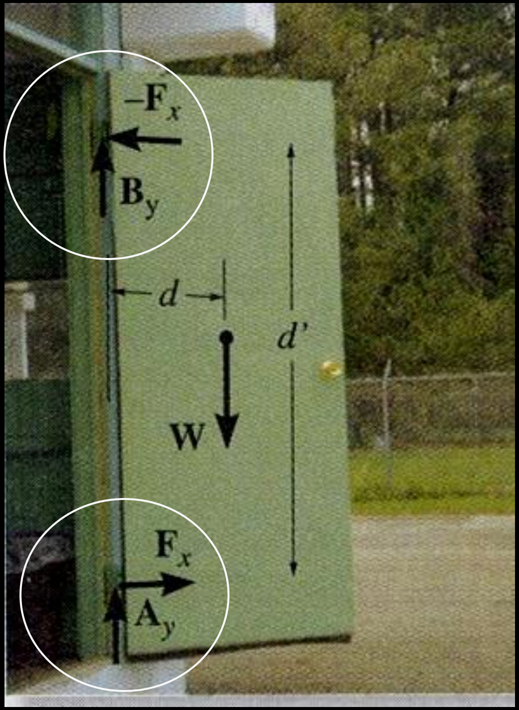


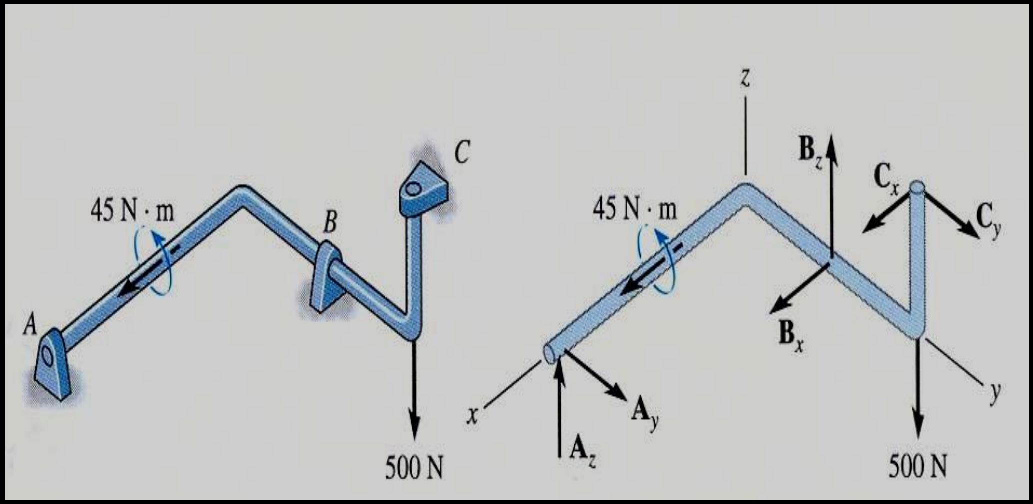


A few examples are shown above. Other support reactions are given in your text book (Table 5-2).

As a general rule, if a support prevents translation of a body in a given direction, then a reaction force acting in the opposite direction is developed on the body. Similarly, if rotation is prevented, a couple moment is exerted on the body by the support.

IMPORTANT NOTE





A single bearing or hinge can prevent rotation by providing a resistive couple moment. However, it is usually preferred to use two or more properly aligned bearings or hinges. Thus, in these cases, only force reactions are generated and there are no moment reactions created.

EQULIBRIUM EQUATIONS IN 3-D

As stated earlier, when a body is in equilibrium, the net force and the net moment equal zero, i.e., $\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$.

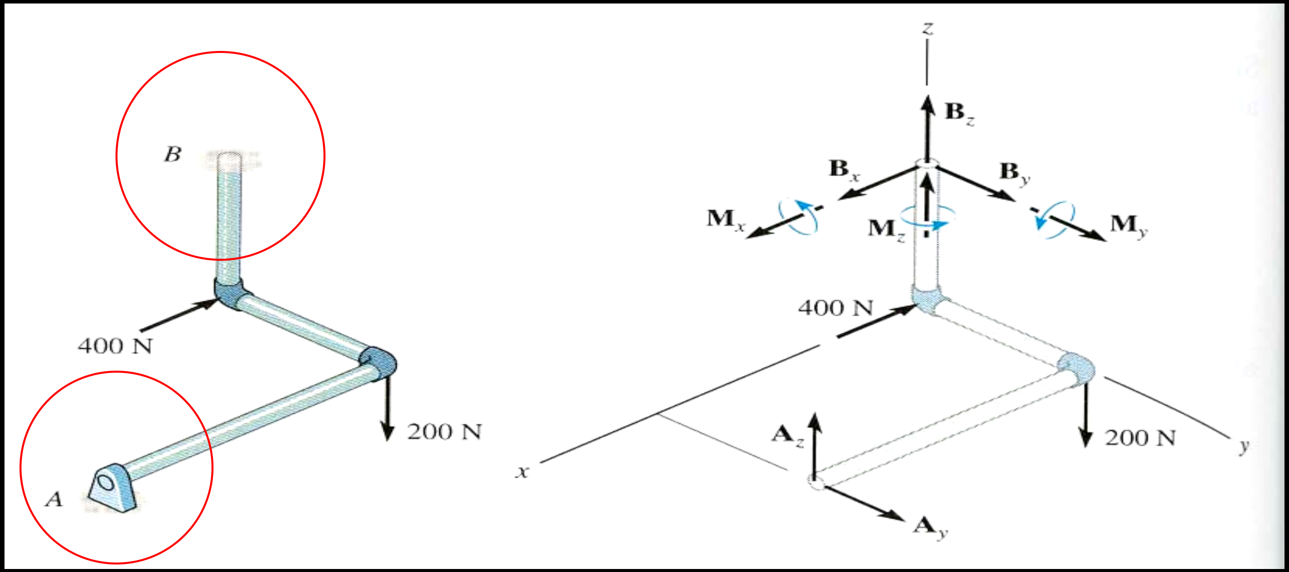
These two vector equations can be written as six scalar equations of equilibrium (EofE). These are

$$\sum F_X = \sum F_Y = \sum F_Z = 0$$

$$\sum M_X = \sum M_Y = \sum M_Z = 0$$

The moment equations can be determined about any point. Usually, choosing the point where the maximum number of unknown forces are present simplifies the solution. Those forces do not appear in the moment equation since they pass through the point. Thus, they do not appear in the equation.

CONSTRAINTS FOR A RIGID BODY

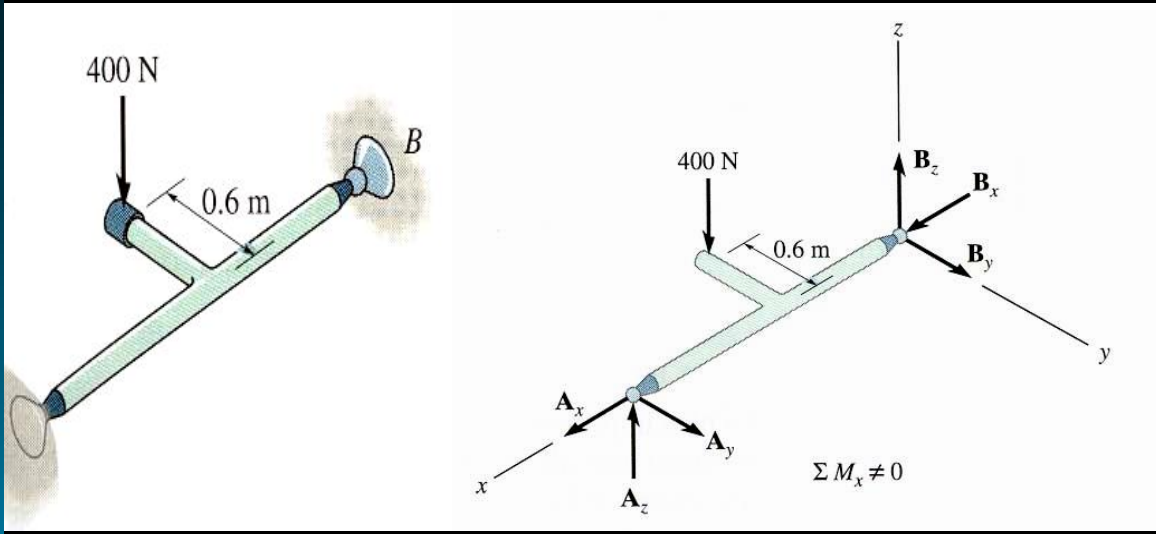


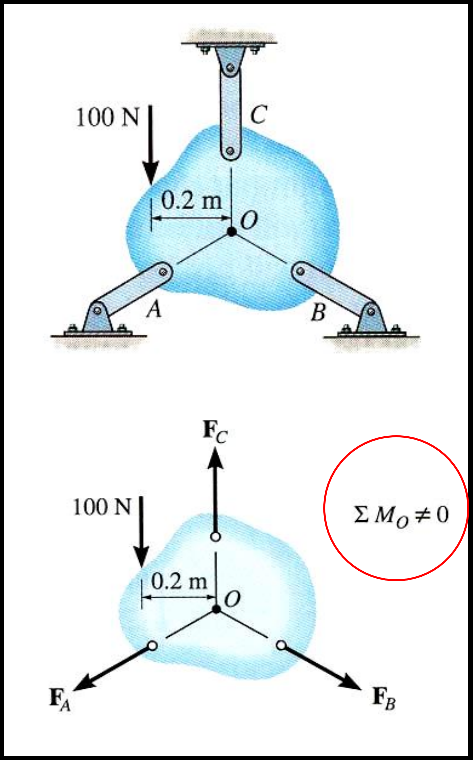
Redundant Constraints: When a body has more supports than necessary to hold it in equilibrium, it becomes statically indeterminate.

A problem that is statically indeterminate has more unknowns than equations of equilibrium.

Are statically indeterminate structures used in practice? Why or why not?

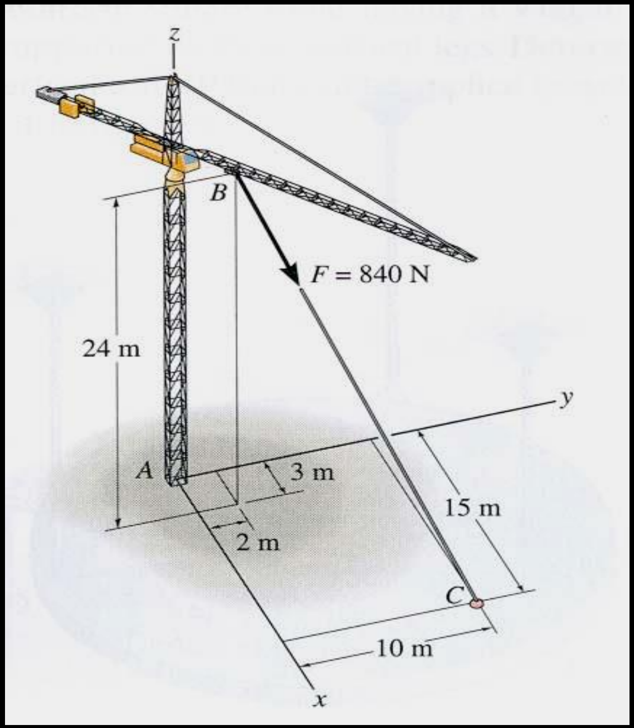
IMPROPER CONSTRAINTS





Here, we have 6 unknowns but there is nothing restricting rotation about the x axis.

In some cases, there may be as many unknown reactions as there are equations of equilibrium. However, if the supports are not properly constrained, the body may become unstable for some loading cases.



EXAMPLE

Given: The cable of the tower crane is subjected to 840 N force. A fixed base at A supports the crane.

Find: Reactions at the fixed base A.

Plan:

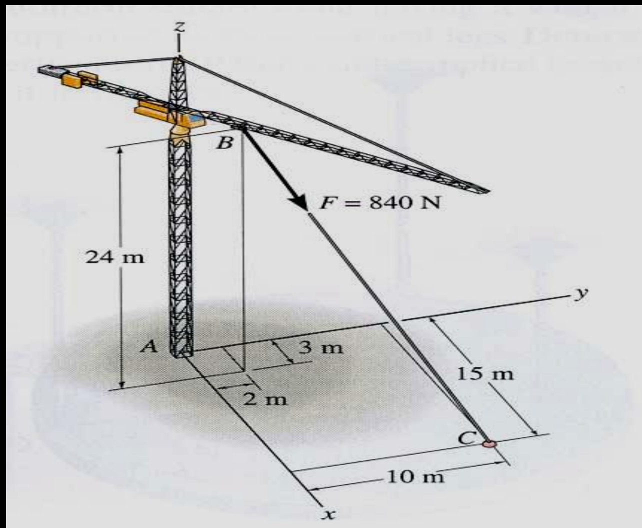
a) Establish the x, y and z axes.

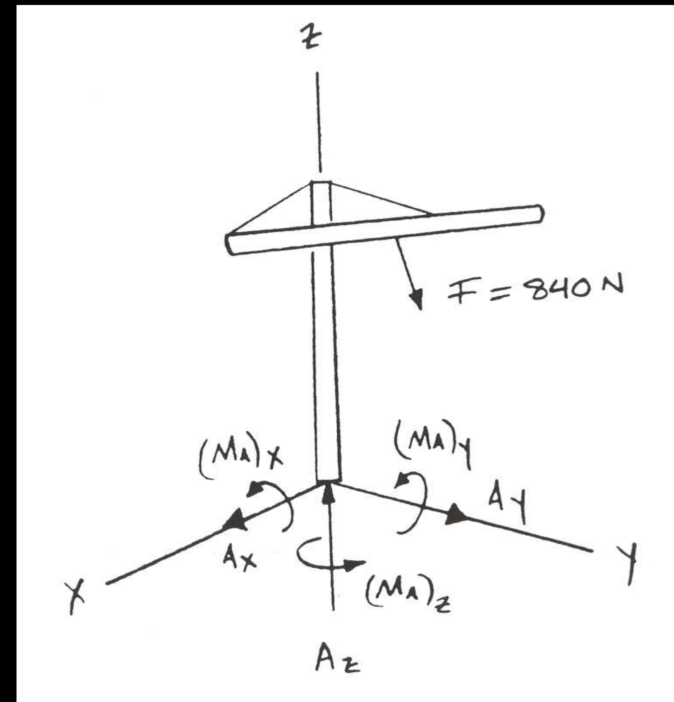
b) Draw a FBD of the crane.

c) Write the forces using Cartesian vector notation.

d) Apply the equations of equilibrium (vector version) to solve for the unknown forces.

EXAMPLE (continued)





$\boldsymbol{r}_{BC}$ = {12 $\boldsymbol{i}$ + 8 $\boldsymbol{j}$ – 24 $\boldsymbol{k}$} m

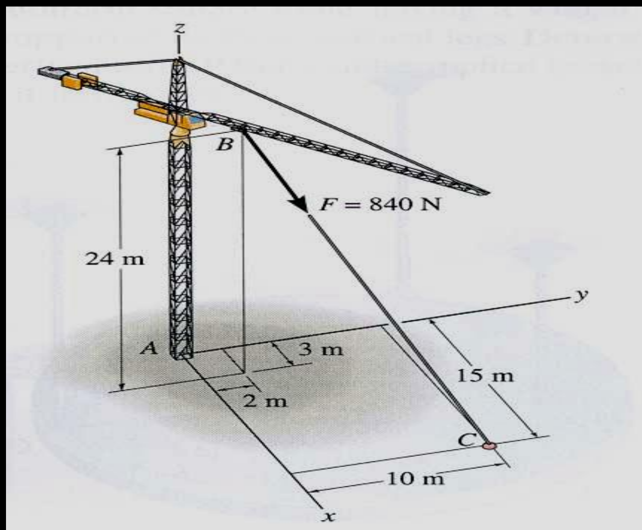
$\boldsymbol{F}$ = F [$\boldsymbol{u}_{BC}$] N

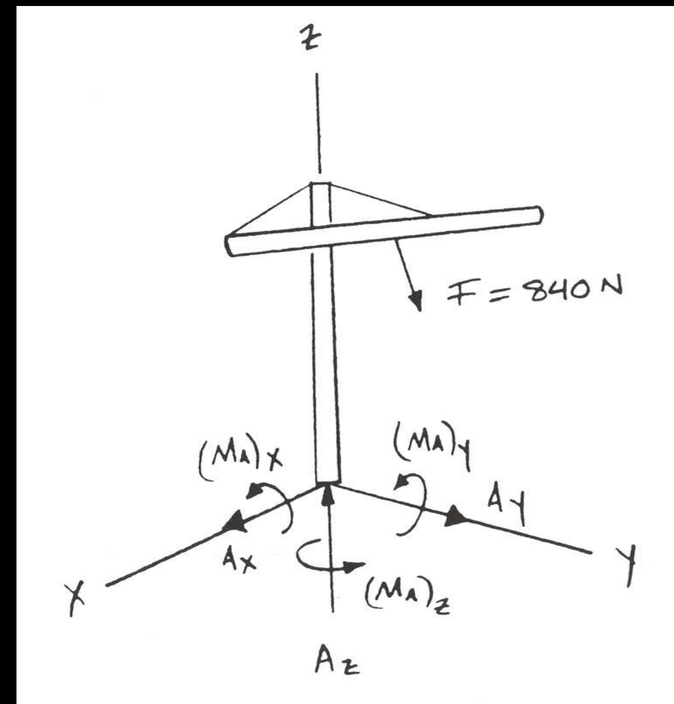
= 840 [12 $\boldsymbol{i}$ + 8 $\boldsymbol{j}$ – 24 $\boldsymbol{k}$] / $(12^2 + 8^2 + (-24^2))^{½}$

= {360 $\boldsymbol{i}$ + 24 $\boldsymbol{j}$ – 720 $\boldsymbol{k}$} N

$\boldsymbol{F}_A$ = {A_X $\boldsymbol{i}$ + A_Y $\boldsymbol{j}$ + A_Z $\boldsymbol{k}$ } N

EXAMPLE (continued)

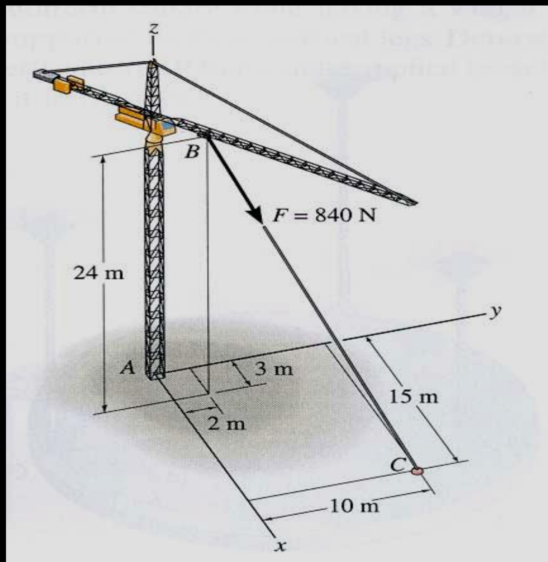




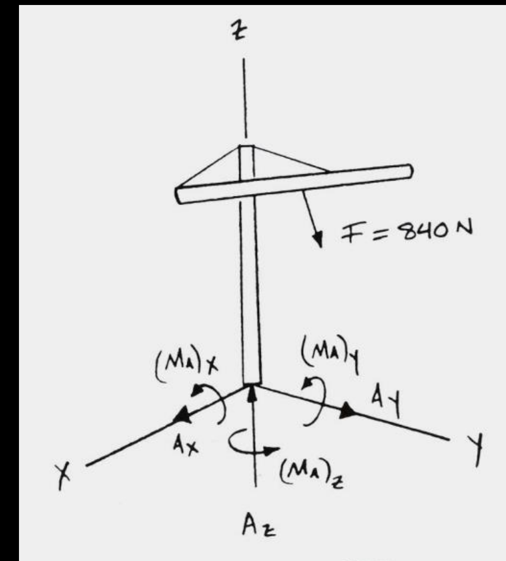
From EofE we get, $\boldsymbol{F} + \boldsymbol{F_A} = 0$

$\{(360 + A_X)\,\boldsymbol{i} + (240 + A_Y)\,\boldsymbol{j} + (-720 + A_Z)\,\boldsymbol{k}\} = 0$

Solving each component equation yields $A_X = -360$ N , $A_Y = -240$ N , and $A_Z = 720$ N.



EXAMPLE (continued)



Sum the moments acting at point A.

$$\sum \mathbf{M} = \mathbf{M_A} + \mathbf{r_{AC}} \times \mathbf{F} = 0$$

$$= M_{AX}\,\mathbf{i} + M_{AY}\,\mathbf{j} + M_{AZ}\,\mathbf{k} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 15 & 10 & 0 \\ 360 & 240 & -720 \end{vmatrix} = 0$$

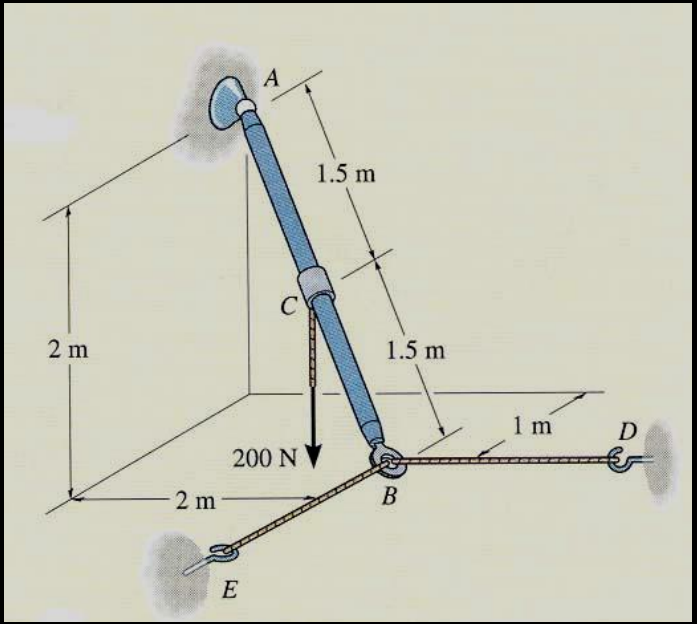
$$= M_{AX}\,\mathbf{i} + M_{AY}\,\mathbf{j} + M_{AZ}\,\mathbf{k} - 7200\,\mathbf{i} + 10800\,\mathbf{j} = 0$$

$$M_{AX} = 7200 \text{ N} \cdot \text{m},\ M_{AY} = -10800 \text{ N} \cdot \text{m, and } M_{AZ} = 0$$

Note: For simpler problems, one can directly use three scalar moment equations, $\sum M_X = \sum M_Y = \sum M_Z = 0$

CONCEPT QUIZ

1. The rod AB is supported using two cables at B and a ball-and-socket joint at A. How many unknown support reactions exist in this problem?



A) 5 force and 1 moment reaction

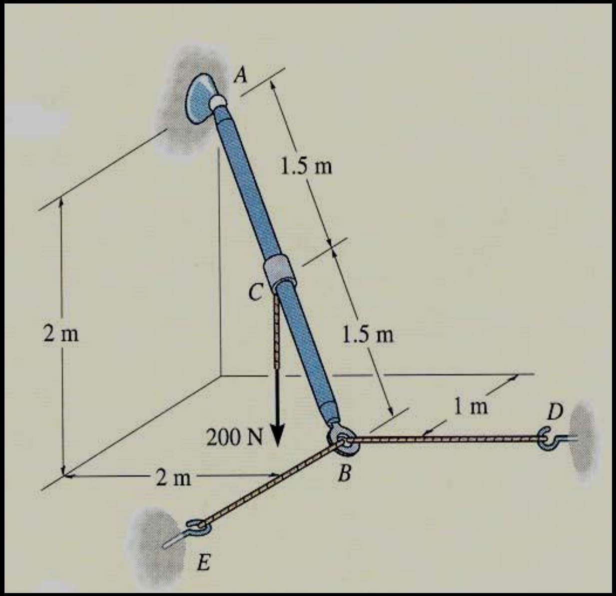
B) 5 force reactions

C) 3 force and 3 moment reactions

D) 4 force and 2 moment reactions

CONCEPT QUIZ (continued)

2. If an additional couple moment in the vertical direction is applied to rod AB at point C, then what will happen to the rod?



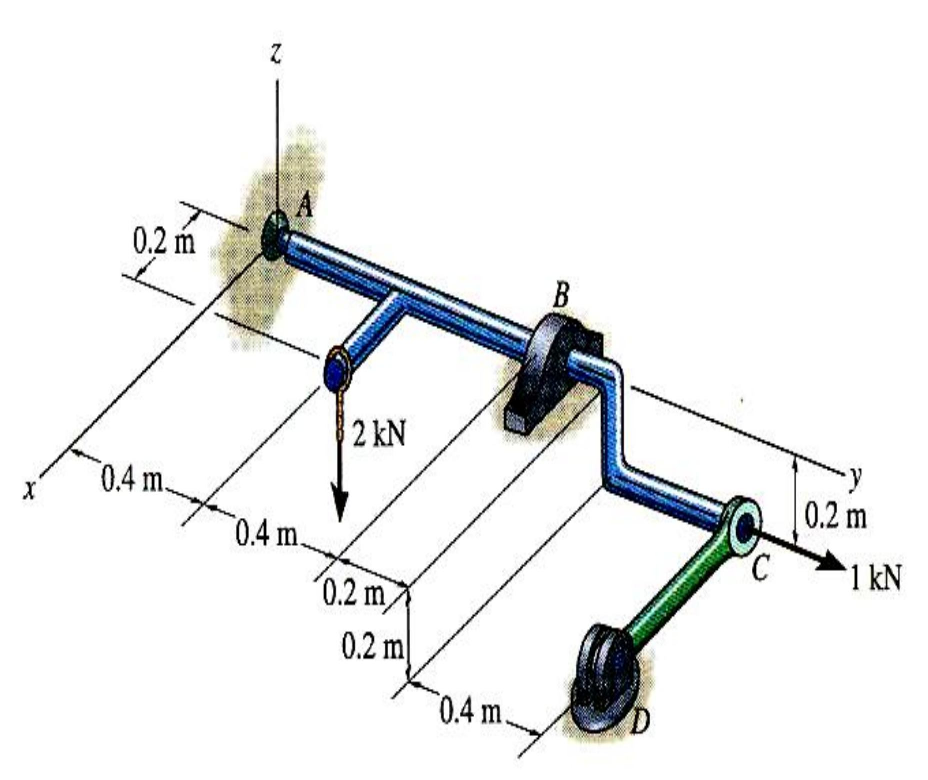
A) The rod remains in equilibrium as the cables provide the necessary support reactions.

B) The rod remains in equilibrium as the ball-and-socket joint will provide the necessary resistive reactions.

C) The rod becomes unstable as the cables cannot support compressive forces.

D) The rod becomes unstable since a moment about AB cannot be restricted.

GROUP PROBLEM SOLVING



Given: A rod is supported by a ball-and-socket joint at A, a journal bearing at B and a short link at C. Assume the rod is properly aligned.

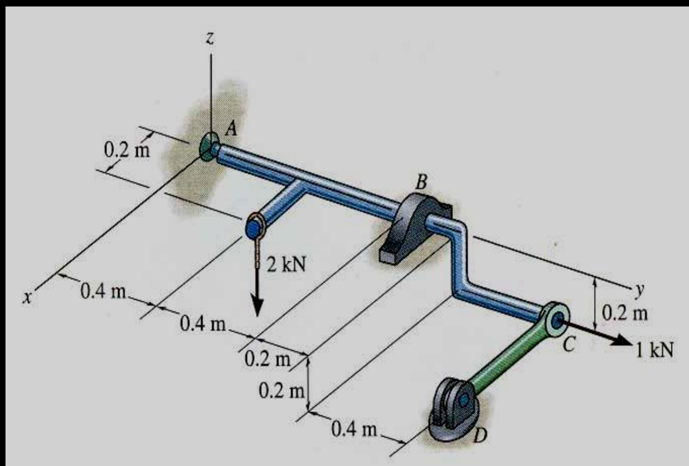
Find: The reactions at all the supports for the loading shown.

Plan:

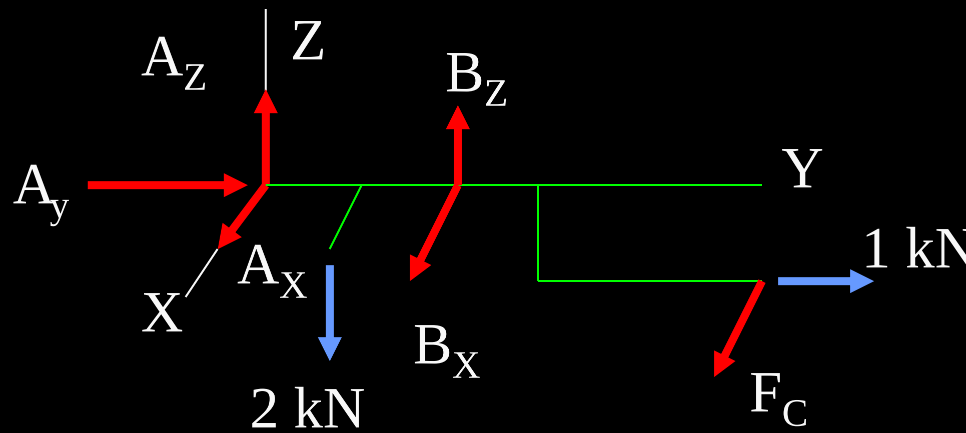
a) Draw a FBD of the rod.

b) Apply scalar equations of equilibrium to solve for the unknowns.

PROBLEM (continued)



A FBD of the rod:



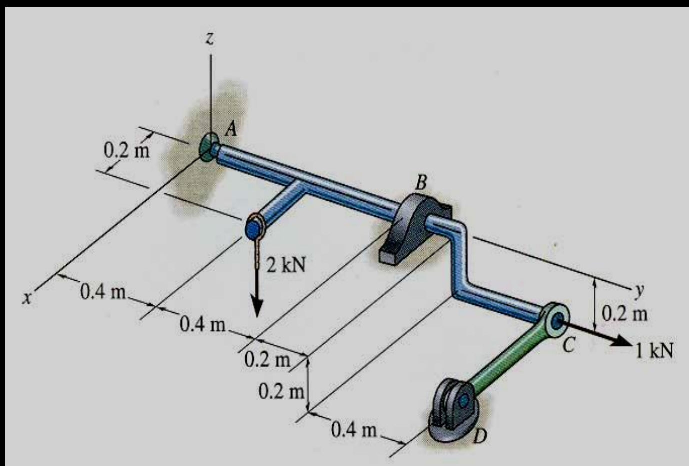
Applying scalar equations of equilibrium in appropriate order, we get

$\sum M_Y = 2\,(0.2) - F_C\,(0.2) = 0\,;$ $F_C = 2\ k\,N$

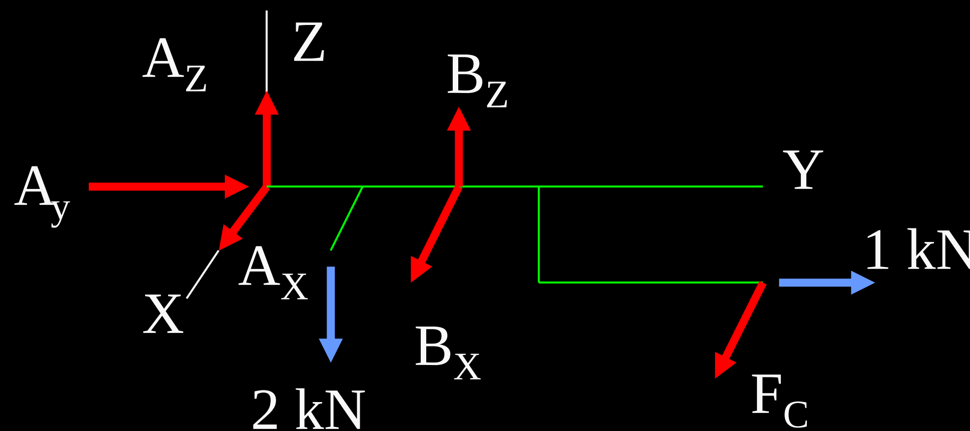
$\sum F_Y = A_Y + 1 = 0\,;$ $A_Y = -1\ k\,N$

$\sum M_Z = -2\,(1.4)\ \ B_X\,(0.8) = 0\,;$ $B_X = -3.5\ kN$

PROBLEM (continued)



A FBD of the rod:



$\sum F_X = A_X - 3.5 + 2 = 0$; $A_X = 1.5$ kN

$\sum M_X = -2(0.4) + B_Z(0.8) + 1(0.2) = 0$; $B_Z = 0.75$ kN

$\sum F_Z = A_Z + 0.75 - 2 = 0$; $A_Z = 1.25$ kN

ATTENTION QUIZ

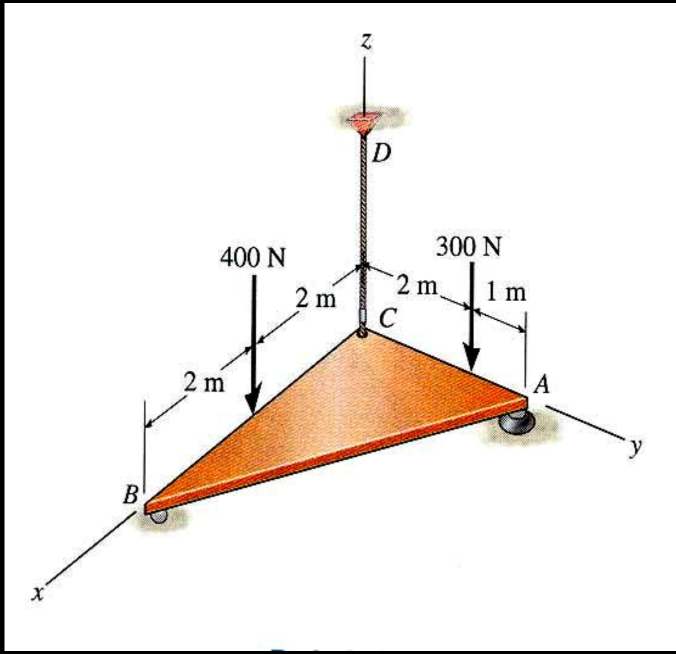
1. A plate is supported by a ball-and-socket joint at A, a roller joint at B, and a cable at C. How many unknown support reactions are there in this problem?

   A) 4 forces and 2 moments

   B) 6 forces

   C) 5 forces

   D) 4 forces and 1 moment



ATTENTION QUIZ

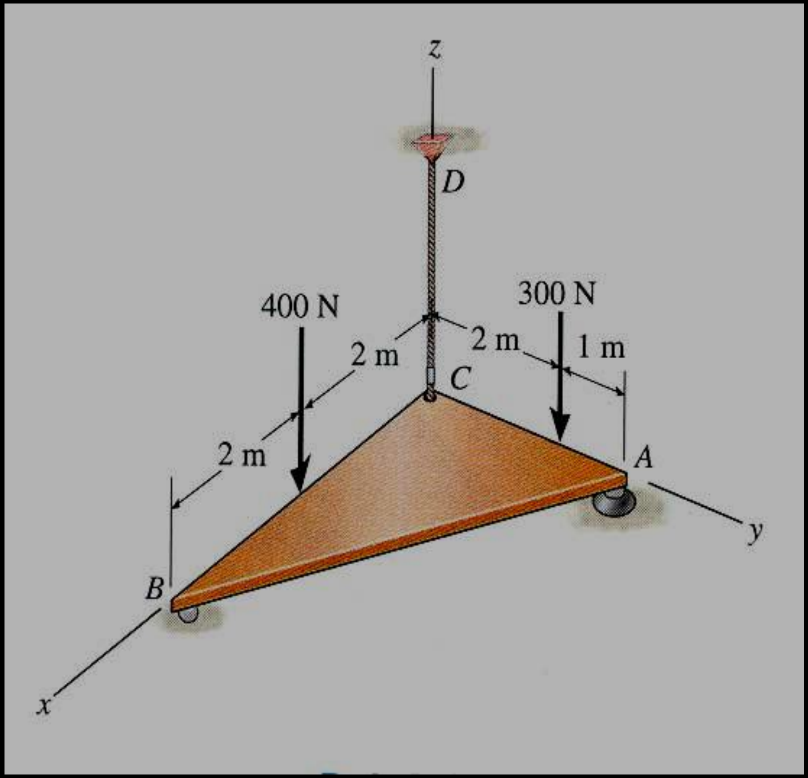
2. What will be the easiest way to determine the force reaction B_Z ?

   A) Scalar equation $\sum F_Z = 0$

   B) Vector equation $\sum \boldsymbol{M_A} = 0$

   C) Scalar equation $\sum M_Z = 0$

   D) Scalar equation $\sum M_Y = 0$



End of the Lecture

Let Learning Continue