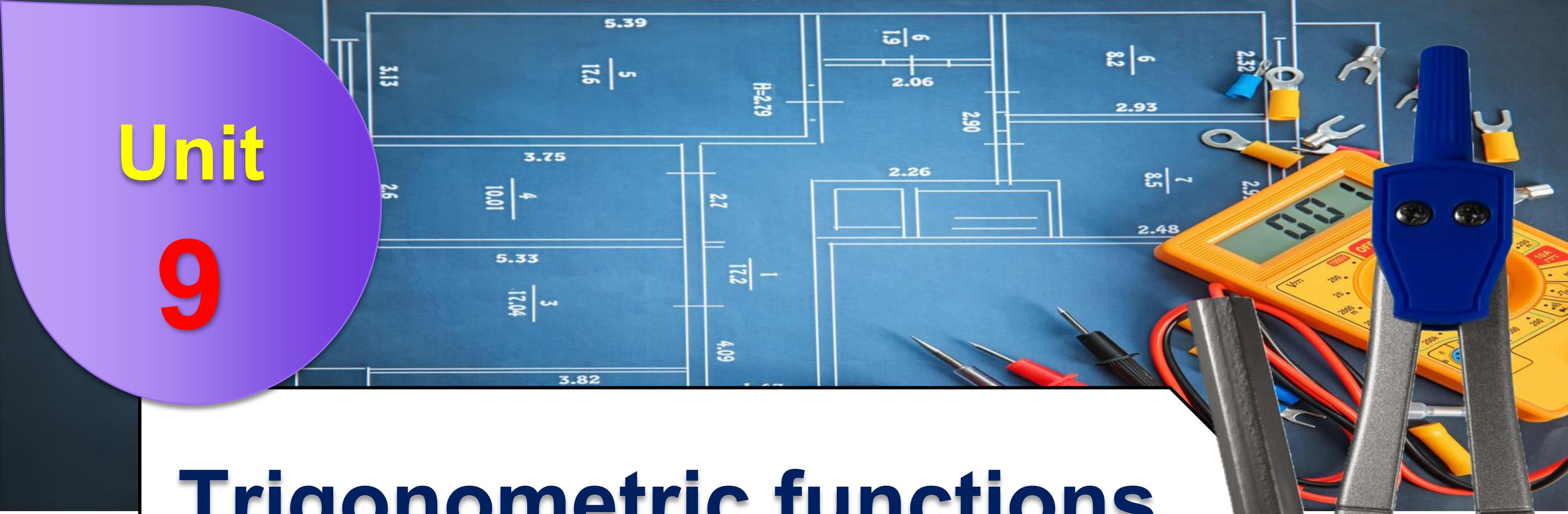
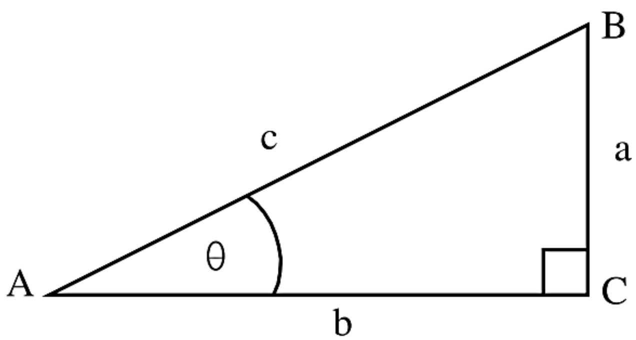


Unit 9

Trigonometric functions

Trigonometric ratios from right triangles



รูปที่ 12.1

A is called the side opposite angle A.

Call b the side adjacent to angle A.

c is called the hypotenuse.

$$\sin A = \frac{\text{ด้านตรงข้ามมุม } A}{\text{ด้านตรงข้ามมุมฉาก}} \qquad \csc A = \frac{\text{ด้านตรงข้ามมุมฉาก}}{\text{ด้านตรงข้ามมุม } A}$$

$$\cos A = \frac{\text{ด้านประชิดมุม } A}{\text{ด้านตรงข้ามมุมฉาก}} \qquad \sec A = \frac{\text{ด้านตรงข้ามมุมฉาก}}{\text{ด้านประชิดมุม } A}$$

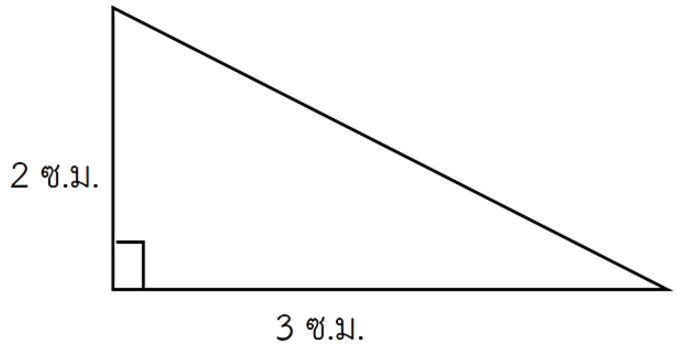
$$\tan A = \frac{\text{ด้านตรงข้ามมุม } A}{\text{ด้านประชิดมุม } A} \qquad \cot A = \frac{\text{ด้านประชิดมุม } A}{\text{ด้านตรงข้ามมุม } A}$$

9

example

From the right triangle, find the angle size 0 (1 decimal place).

วิธีทำ



รูปที่ 12.5

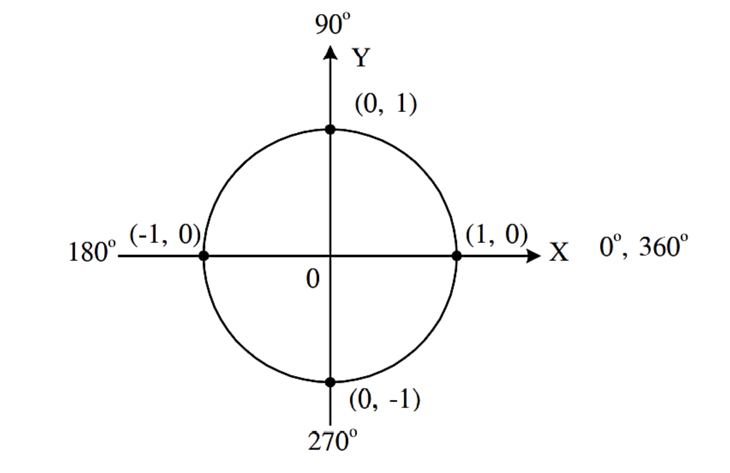
จากรูป $\tan\theta = \frac{2}{3}$

$= 0.6667$

$= 33.7°$

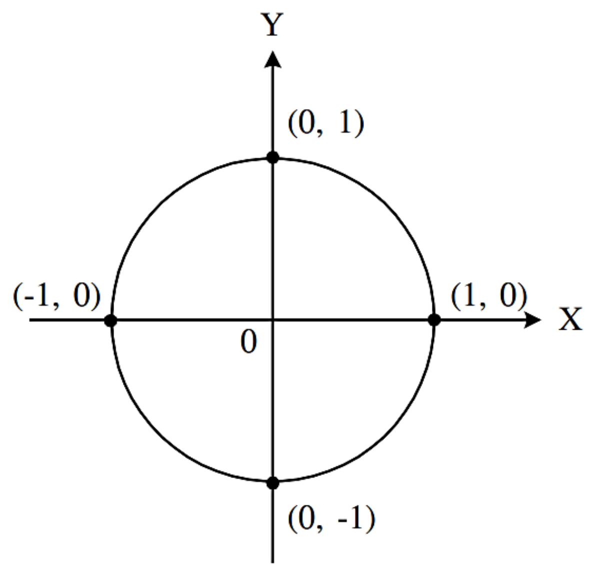
Finding Trigonometric Functions for Angles Around the Center

A unit circle is a circle with a radius of 1 unit. The center is at the origin. Orthogonal coordinate plane



รูปที่ 12.6

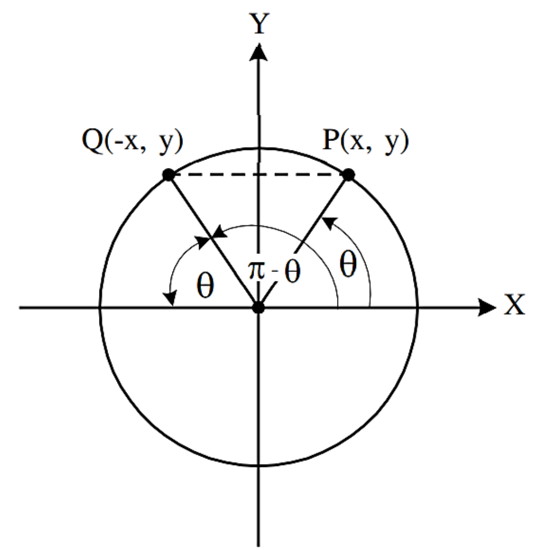
Evaluating trigonometric functions of angles with ends on the x and y axes.



รูปที่ 12.8

Relationship of trigonometric function values of angles in the 2nd quadrant to the 1st quadrant.

If θ is an angle in the first quadrant, then the angle in the second quadrant is written in general form as π- 0 or 180° - θ.



รูปที่ 12.10

that is

$\sin(\pi - \theta) = \sin(180^\circ - \theta) = \sin\theta$

$\cos(\pi - \theta) = \cos(180^\circ - \theta) = -\cos\theta$

$\tan(\pi - \theta) = \tan(180^\circ - \theta) = -\tan\theta$

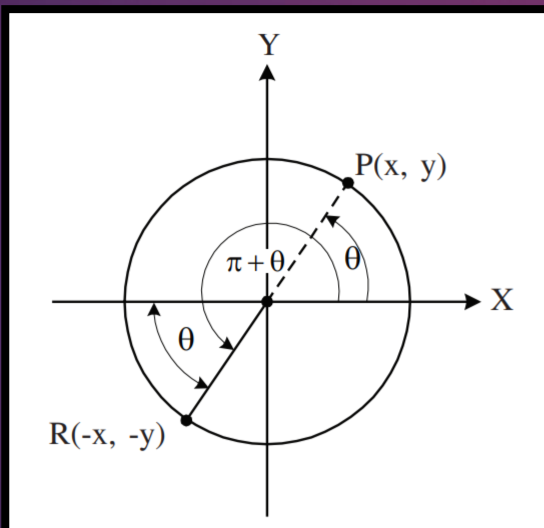
$\csc(\pi - \theta) = \csc(180^\circ - \theta) = \csc\theta$

$\sec(\pi - \theta) = \sec(180^\circ - \theta) = -\sec\theta$

co πt$(\pi - \theta) = \cot(180^\circ - \theta) = -\cot\theta$

Relationship of trigonometric function values of angles in the 3rd quadrant and the 1st quadrant.

If θ is an angle in the 1st quadrant, then the angle in the 3rd quadrant is written in general form as (π+ 0) or (180° + θ).



รูปที่ 12.11

that is

$\sin(\pi + \theta) = \sin(180^\circ + \theta) = -\sin\theta$

$\cos(\pi + \theta) = \cos(180^\circ + \theta) = -\cos\theta$

$\tan(\pi + \theta) = \tan(180^\circ + \theta) = \tan\theta$

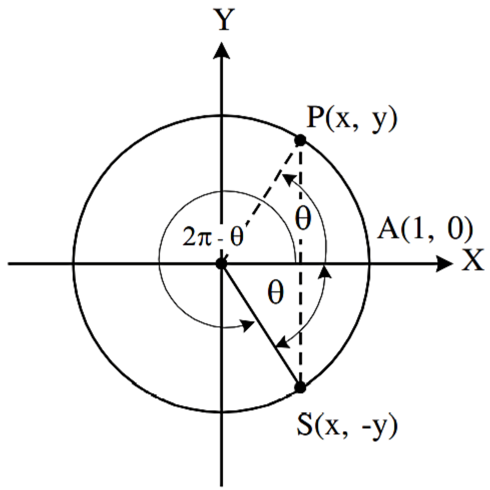
$\csc(\pi + \theta) = \csc(180^\circ + \theta) = -\csc\theta$

$\sec(\pi + \theta) = \sec(180^\circ + \theta) = -\sec\theta$

$\cot(\pi + \theta) = \cot(180^\circ + \theta) = \cot\theta$

Relationship of trigonometric function values of angles in the 4th quadrant and the 1st quadrant.

If θ is an angle in the 1st quadrant, then the angle in the 4th quadrant is written in general form as (2π - θ) or (360° - θ).



รูปที่ 12.12

that is

$\sin(2\pi + \theta) = \sin(360^\circ + \theta) = -\sin\theta$

$\cos(2\pi + \theta) = \cos(360^\circ + \theta) = \cos\theta$

$\tan(2\pi + \theta) = \tan(360^\circ + \theta) = -\tan\theta$

$\csc(2\pi + \theta) = \csc(360^\circ + \theta) = -\csc\theta$

$\sec(2\pi + \theta) = \sec(360^\circ + \theta) = \sec\theta$

$\cot(2\pi + \theta) = \cot(360^\circ + \theta) = -\cot\theta$

example

9

Find the following values.

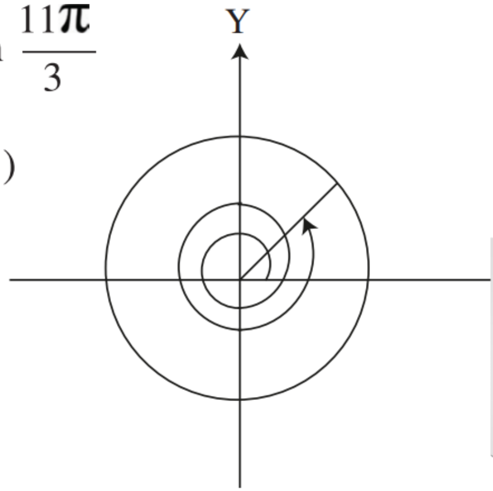
(1) $\sin 750^\circ$ (2) $\cos 870^\circ$ (3) $\tan \frac{11\pi}{3}$

วิธีทำ

(1) $\sin 750^\circ = \sin(2 \cdot 360^\circ + 30^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$



รูปที่ 12.15

รูปที่ 12.15

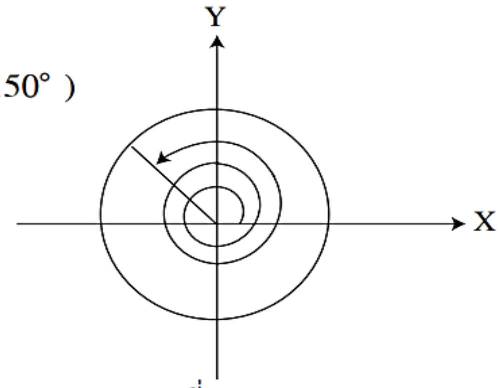
(2) $\cos 870^\circ = \cos(2 \cdot 360^\circ + 150^\circ)$

$= \cos 150^\circ$

$= \cos(180^\circ - 30^\circ)$

$= -\cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$

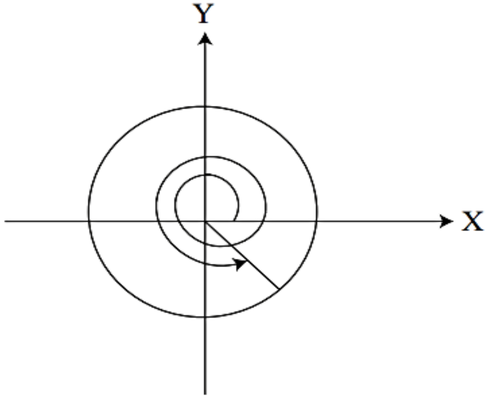


รูปที่ 12.16

(3) $\tan \frac{11\pi}{3} = \tan(2 \cdot 2\pi - \frac{\pi}{3})$

$= -\tan \frac{\pi}{3}$

$= -\sqrt{3}$



รูปที่ 12.17

Operations of trigonometric functions

ผลบวกและผลต่างของมุม 2 มุม

1. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
2. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
3. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
4. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
5. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Multiplication of trigonometry

1. $\sin A \cos B = \sin (A + B) + \sin (A - B)$
2. $\cos A \sin B = \sin (A + B) - \sin (A - B)$
3. $\cos A \cos B = \cos (A + B) + \cos (A - B)$
4. $\sin A \cos B = \cos (A - B) - \cos (A + B)$

การบวกและการลบตรีโกณมิติ

1. $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

2. $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

3. $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

4. $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

example

9

จงหาค่า sin 105° cos 15°

วิธีทำ

จาก $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$$\sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$\sin 105^{\circ}\cos 15^{\circ} = \frac{1}{2}\left(\sin(105^{\circ}+15^{\circ}) + \sin(105^{\circ}-15^{\circ})\right)$$

$$= \frac{1}{2}\left(\sin 120^{\circ} + \sin 90^{\circ}\right)$$

$$= \frac{1}{2}\left(\sin 60^{\circ} + \sin 90^{\circ}\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} + 1\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}+2}{2}\right)$$

$$\sin 105^{\circ}\cos 15^{\circ} = \frac{\sqrt{3}+2}{4}$$